Hexagonal Close Pack Mesh for Fluid Dynamics

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Although regular orthogonal mesh is usually used in numerical relativity, much isotropic mesh is basically preferable for complex fluid motions from the points of numerical accuracy and stability. Hexagonal close pack(HCP) mesh is thought to be an ideal for those purposes. However, HCP has never been used so far since it is difficult to define HCP with data types existing programming language such as C/C++ provides. In this presentation, I propose novel coordinate system suitable for it. Furthermore, Voronoi tessellation and mesh refinement is described.

Keywords: Hexagonal Close Pack; Coordinate System; Mesh Refinement; Voronoi tessellation; Finite Volume Method;

1. Twisted Cartesian Coordinate

In case of HCP mesh, three principal axes of unit cell strongly skew. As a result, the data arrays defining HCP span oblong so much. Here I introduce twisted Cartesian coordinate to make HCP implemented easily and efficiently with common programming language. Keep in mind, hereafter I treat tri-layered type (i.e. Face-Centered Cubic) of HCP from the point of symmetry. Figure 1 describes some details of its procedure.

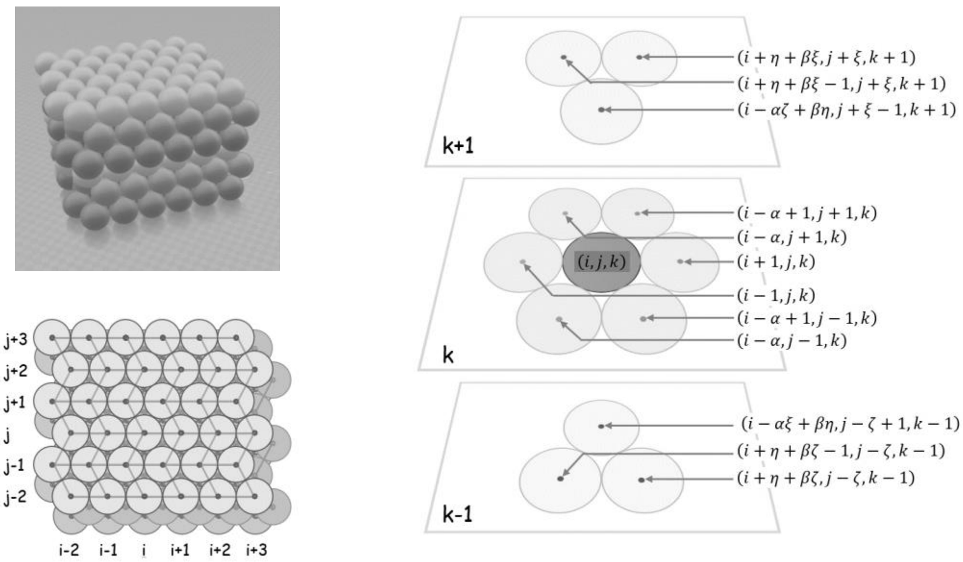

Fig. 1. Spatial configuration (top-left), arrangement of grid points (bottom-left) and a stencil of nearest neighbors (right).

Definition of symbols in the figure follows.

$$\xi \equiv \lfloor (k\%3)/2 \rfloor = \begin{cases} 1 & if\ k\%3 = 2 \\ 0 & otherwise \end{cases} \qquad \alpha \equiv j\%2 = \begin{cases} 1 & if\ j\%2 = 1 \\ 0 & otherwise \end{cases}$$
$$\eta \equiv \lfloor ((k+1)\%3)/2 \rfloor = \begin{cases} 1 & if\ k\%3 = 1 \\ 0 & otherwise \end{cases} \qquad \beta \equiv (j+1)\%2 = \begin{cases} 1 & if\ j\%2 = 0 \\ 0 & otherwise \end{cases} \qquad .(A.1).$$
$$\zeta \equiv \lfloor ((k+2)\%3)/2 \rfloor = \begin{cases} 1 & if\ k\%3 = 0 \\ 0 & otherwise \end{cases}$$

2. Voronoi Tessellation

In computational fluid dynamics, finite volume method is frequently used as a versatile mean to calculate flow fields over complicated geometry. It gives a change of quantity in a volume by summing flux across surfaces. Accordingly, to employ FCC for finite volume method, a partition of space without any opening is required, Voronoi tessellation is available for this purpose. In this case, rhombic dodecahedron serves as a partitioned volume. Although it is not a platonic solid, it has good symmetry since its surface is comprised of twelve identical rhomboids. In addition, it is noteworthy that surface-to-volume ratio of rhombic dodecahedron is just half of cube and same to sphere.

3. Mesh Refinement

Recursive mesh refinement of HCP is possible through the steps in what follows. First, make all voxels shrunk half in length around its center, then tile generated space between voxels in each layer with additional voxels. Finally, insert reciprocal layers to fit void between existing layers. Current interpolation procedure of data from parents to a child still remains asymmetric partly and left room to be improved. Contrary, agglomeration procedure of children to a parent is straightforward.

References

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