Late time cosmic acceleration with non-minimally coupled gravity

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In current cosmic scenario, an accelerated expansion era is being reported by various observations. The reasoning of such scenario is still unknown, the presence of an unknown energy component is seen which is named as dark energy (DE). There are various approaches to discuss the existence of DE and present cosmic acceleration. One of such attempts is the modification of Einstein's gravity, here we attempt to explore this problem in the framework of modified gravity based on non-minimal matter-geometry coupling. $f(R,T,Q)$ modified theory is chosen (where R is the Ricci Scalar, T is the trace of energy-momentum tensor (EMT) T_{uv} and $Q = R_{uv}T^{uv}$ is interaction of EMT $T_{\mu\nu}$ and Ricci Tensor R_{uv}) to address this problem. We formulate the dynamical equations in the background of Friedmann-Lernaitre-Robertson-Walker (FLRW) model and find the result of non-conserved EMT using the divergence of the field equations. In this scenario motion of test particles is non-geodesic and an extra force orthogonal to four-velocity of the particle is present due non-minimal coupling. We applied this result to find an expression for energy density ρ for particular choice of Lagrangian. Furthermore, we discuss the energy bound on the model parameters and discuss the late time cosmic acceleration for best suitable parameters in accordance with recent observations.

1. introduction

Currently our universe is experiencing an accelerated expansion phase and multiple astrophysical researches have been conducted to observe this cosmic scenario. It is highly assumed and considered that this cosmic acceleration is the consequence of an anonymous energy named as DE[1]. Antagonistic to the gravitational pull, the DE is expanding the universe by having a negative pressure which is completely opposite to the ordinary matter. Many attempts have been made to unveil the reason for accelerated cosmic expansion. The major finding[2] enlists DE as the major candidate with overall contribution of 68.3% the other significant 26.8% contribution is from Dark matter despite its elusive and un-explored nature. Baryon, is the major part of visible cosmos which accounts for 4.9% among cosmic ingredients. Despite tremendous researches and observations, late time cosmic acceleration is still a significant as well as challenging area for cosmologists. However, attention is attached to the confirmation through measurements from temperature anisotropies of the existence of DE as puzzling cosmic ingredient with reference to cosmic acceleration by cosmic microwave background radiations (CMBR)[2], baryon acoustic oscillations (BAO)[3], large scale structure (LSS)[4], weak lensing[5] and most recent plank's data[6]. Moreover, to explain the behavior of DE several theoretical models are proposed like phantom[7], quintessence[8] and fluids with anisotropic EoS[9]. In Λ cold dark matter (ΛCDM) model, role of DE in GR is played by Λ. Yet the origin of cosmological constant Λ is still under question and Λ has two well-known problems known as coincidence and fine-tuning. To express the characteristics of DE, the

EoS is proposed as $\omega_{DE} = \frac{p_{DE}}{\rho_{DE}}$ (the ratio of the pressure to the energy density of DE). The equation of state is evaluated by using FLRW spcae time and considering cosmological principle. ω_{DE} is a constant and equal to -1 in ΛCDM model whereas in quintessence model ω_{DE} is dynamical quantity and $-1 < \omega_{DE} < \frac{-1}{3}$. Moreover ω_{DE} varies with time and $\omega_{DE} < -1$ in phantom model. The preliminary step is to substitute Einstein Hilbert term by scalar curvature and it results in the formation of $f(R)$ theory [10]. In this theory, general non-linear function f depends on the Ricci scalar and if we replace this generic function f by $f \equiv R - 2\Lambda$ then we will get the classic ΛCDM model. This theory is also interesting due to the fact that for a specific Brans Dicke (BD) parameter [11] it develops correspondence with the BD theory. This coupling is also further constructed in $f(R)$ theory [12,13].

The nonminimal coupling further lead to non-conserved matter energy-momentum tensor(EMT) which results in deviation of test particles from geodesic motion [14]. In [15], Harko generalized this non-minimal coupling by introducing a function of matter Lagrangian. Later Wu [16] further extended this work by studying few forms of curvature components and forming the thermodynamic laws. Harko along with the contributions of Lobo [17] proposed another induced form of $f(R)$ by involving curvature matter coupling incorporating matter langrangian $\mathcal{L}_m$ and defined generic function $f(R, \mathcal{L}_m)$. In [18], Sharif and Zubair discussed the non-equilibrium thermodynamics in $f(R, \mathcal{L}_m)$ gravity, and develop constraints on two specific gravitational models $f(R, \mathcal{L}_m) = \lambda \exp\left(\frac{1}{2\lambda}R + \frac{1}{\lambda}\mathcal{L}_m\right)$ and $f(R, \mathcal{L}_m) = \alpha R + \beta R^2 + \gamma \mathcal{L}_m$ to secure the validity of GSLT in this theory.

The selection of matter Lagrangian density has an issue in modified theories, specifically for those which involve nonminimal coupling with matter Lagrangian. For the natural conservation of matter we are restricted to take matter Lagrangian as $\mathcal{L}_m = p$ then extra force will be vanished [19] or for the sake of effective nonminimal coupling we can also take $\mathcal{L}_m = -\rho$ [20]. Due to remarkable attentions towards modified theories the efficacy of thermodynamics laws in $f(R, T)$ theory have been explored by Sharif and Zubair [21] and it is concluded that the equilibrium thermodynamic laws cannot be achieved due to matter geometry interaction. Attempts to reconstruct $f(R, T)$ Lagrangian has also been made under various considerations like the family of holographic DE models by supposing the FLRW universe [22], considering an auxiliary scalar field [23] and anisotropic solutions [24]. Jamil et.al [25] worked on the reconstruction of cosmological models and they showed that the dust fluid reproduce ΛCDM, Einstein static universe and de sitter Universe. Alvarenga et al. [26] discussed the development of matter density perturbations in this theory and they presented the required constraints to get the standard continuity equation in $f(R, T)$ gravity. In [27], authors reconstructed cosmological models by applying additional constraints for the conserved EMT and studied the stability of the constructed models. Furthermore, the dynamical systems in $f(R, T)$ theory were explored by Shabani and Farhoudi [28] that resulted in the development of a vast scale of passable cosmological solutions. Other cosmic issues including compact stars, wormholes and gravitational instability of collapsing stars have been discussed in literature [29].

Lately, the non-minimal coupling of the EMT and Ricci tensor is introduced, resulting in the modified yet more complicated theory known as $f(R,T,Q)$ gravity [30,31]. Due to complicated nonminimal matter-geometry coupling EMT is generally non-conserved and additional force is there. Therefore, it proposes a vast range to explore different cosmic features as thermodynamics properties have already been studied by Sharif and Zubair [32]. E.H.Baffou et al. [33] discussed the stability of desitter and power law solution by using perturbation scheme for particular models. In this paper we intend to discus the cosmological evolution in $f(R,T,Q)$ theory, which is based on more general matter-geometry coupling. We pick a particular model of the form $f(R,T,Q) = R(1+\alpha Q)$, and solve the matter conservation equation to find the explicit expression of energy density. Evolution of EoS parameter ω_{eff} and deceleration parameter is discussed employing the power law cosmology. This manuscript is organized of the form: In Sec. II, a brief introduction of $f(R,T,Q)$ theory and its general formalism of field equations is presented. Section III is devoted to particular model in this theory where, we present the expressions for ρ_{eff}, p_{eff} and ω_{eff}. In section IV we constrain the model parameters using the energy bounds. Section V, concludes our discussion.

2. $f(R,T,Q)$ Gravity

$f(R,T,Q)$ gravity is the most generic gravity theory among other modified gravities like $f(R)$ and $f(R,T)$ and this theory is very effective for non-minimal matter-curvature coupling. The action of this complicated theory takes the following form [30,31]

$$\mathcal{A} = \frac{1}{2\kappa^2}\int dx^4\sqrt{-g}\,[f(R,T,R_{\mu\nu}T^{\mu\nu}) + \mathcal{L}_m], \tag{1}$$

where $\kappa^2 = 8\pi G$, $f(R,T,Q)$ is a general function which depends on three components, R, T, product of the EMT $T^{\mu\nu}$ to Ricci tensor $R_{\mu\nu}$, and $\mathcal{L}_m$ shows the matter Lagrangian. The EMT for matter is defined as

$$T_{\mu\nu} = g_{\mu\nu}\mathcal{L}_m - \frac{2\partial\mathcal{L}_m}{\partial g^{\mu\nu}}. \tag{2}$$

The field equations in this modified gravity can be found as

$$R_{\mu\nu}f_R - \{\frac{1}{2}f - \mathcal{L}_m f_T - \frac{1}{2}\nabla_\gamma\nabla_\delta(f_Q T^{\gamma\delta})\}g_{\mu\nu} - (\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box)f_R + \frac{1}{2}\Box(f_Q \tag{3}$$

$$T_{\mu\nu}) + 2f_Q R_{\gamma(\mu}T^\gamma_{\nu)} - \nabla_\gamma\nabla_{(\mu}[T^\gamma_{\nu)}f_Q] - G_{\mu\nu}\mathcal{L}_m f_Q - 2(f_Q R^{\gamma\delta} + f_T g^{\gamma\delta})\frac{\partial^2\mathcal{L}_m}{\partial g^{\mu\nu}\partial g^{\gamma\delta}}$$

$$= (1 + f_T + \frac{1}{2}Rf_Q)T_{\mu\nu}. \tag{4}$$

The subscripts shows the derivatives w. r. t R,T,Q, and box function defined as $\Box = \nabla^\delta\nabla_\delta$, ∇_μ represent covariant derivative. If we will choose the particular form of Lagrangian then Equation (3) can be shifted towards the well known field equations in $f(R)$ and $f(R,T)$ theories. Field equation (3) can be expressed into the form of effective Einstein field equation (EFE) as

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = T^{eff}_{\mu\nu}. \tag{5}$$

This effective form of EFE is identical to GR's equations. Here, $T^{eff}_{\mu\nu}$, the effective EMT in $f(R,T,Q)$ gravity is found to be as

$$T^{eff}_{\mu\nu} = \frac{-1}{f_Q\mathcal{L}_m - f_R}[(\frac{1}{2}Rf_Q + f_T + 1)T_{\mu\nu} - \frac{1}{2}(Rf_R - f) - \mathcal{L}_m f_T - \frac{1}{2}\nabla_\gamma\nabla_\delta(f_Q T^{\gamma\delta})$$
$$g_{\mu\nu} - (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)f_R - \frac{1}{2}\Box(f_Q T_{\mu\nu})2f_Q R_{\gamma(\mu}T^{\gamma}_{\nu)} + \nabla_\gamma\nabla_{(\mu}[T^{\gamma}_{\nu)}f_Q] + 2(f_T g^{\gamma\delta}$$
$$+ f_Q R^{\gamma\delta})\frac{\partial^2\mathcal{L}_m}{\partial g^{\mu\nu}\partial g^{\gamma\delta}}]. \tag{6}$$

We prevail the following dynamical equation after using the covariant divergence for the modified Einstein equations

$$\nabla^\mu T_{\mu\nu} = \frac{2}{2(1+f_T) + Rf_Q}[\nabla_\mu(f_Q R^{\gamma\mu}T_{\gamma\nu}) + \nabla_\nu(\mathcal{L}_m f_T) - \frac{1}{2}(f_Q R_{\sigma\zeta} f_T g_{\sigma\zeta})\nabla_\nu$$
$$T^{\sigma\zeta} - G_{\mu\nu}\nabla^\mu(f_Q\mathcal{L}_m) - \frac{1}{2}[\nabla^\mu(Rf_Q) + 2\nabla^\mu f_T]T_{\mu\nu}]. \tag{7}$$

It is important to see that any modified theory which involve nonminimal coupling between geometry and matter does not obey the ideal continuity equation. This complicated theory $f(R,T,Q)$ also involves this type of nonminimal coupling so it also deviate from standard behavior of continuity equation. Here, non-minimal coupling between matter-geometry induces extra force acting on massive particles, whose equation of motion is given by[31]

$$\frac{d^2x^\lambda}{ds^2} + \Gamma^\lambda_{\mu\nu}u^\mu u^\nu = f^\lambda,$$

where

$$f^\lambda = \frac{h^{\lambda\nu}}{(\rho+p)(1+2f_T+Rf_{RT})}\Big[(f_T + Rf_{RT})\nabla_\nu\rho - (1+3f_T)\nabla_\nu p - (\rho+p)f_{RT}$$
$$R^{\sigma\rho}(\nabla_\nu h_{\sigma\rho} - 2\nabla_\rho h_{\sigma\nu}) - f_{RT}R_{\sigma\rho}h^{\sigma\rho}\nabla_\nu(\rho+p)\Big]. \tag{8}$$

This is shown that the impact of non-minimal coupling is always present independent of the choice matter Lagrangian, the additional force does not disappear even if we set the matter Lagrangian as $\mathcal{L}_m = p$ as compared to the results presented in[34]. In[31], authors also presented the Lagrange multiplier approach and found the conservation of matter EMT. Moreover, if one eliminates the dependence of Q, it results in divergence equation of $f(R,T)$ theory as given below

$$\nabla^\gamma T_{\gamma\delta} = \frac{f_T}{1-f_T}\left[(\Theta_{\gamma\delta} + T_{\gamma\delta})\nabla^\gamma ln f_T - \frac{1}{2}g_{\gamma\delta}\nabla^\gamma T + \nabla^\gamma\Theta_{\gamma\delta}\right].$$

In [26], Alvarenga et al.shown that choice of a specific model within these theories can guarantee the conservation of EMT and continuity equation is valid for the model $f(R,T) = f_1(R) + f_2(T)$, where $f_2(T) = \alpha T^{\frac{1+3\omega}{2(1+\omega)}} + \beta$. In this manuscript, we are interested to evaluate the role of non-minimal coupling in cosmic evolution so we opt the nonconserved dynamical equation and evaluate necessary parameters.

We select the isotropic and homogenous flat FLRW metric defined as $ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$, where $a(t)$ represents the scale factor. The effective energy density and pressure for this metric is found to be the components of $T^{eff}_{\mu\nu}$, which assumes the form of perfect fluid as

$$T_{\mu\nu} = (p + \rho)u_\mu u_\nu - pg_{\mu\nu} \tag{9}$$

where p represent pressure, ρ for proper density and u_μ is for 4-velocity. In FLRW background, ρ_{eff} and p_{eff} can be found as

$$\rho_{eff} = \frac{-1}{f_Q\mathcal{L}_m - f_R}[\rho - (\mathcal{L}_m - \rho)f_T - \frac{1}{2}(Rf_R - f) - 3H\partial_t f_R - \frac{3}{2}(3H^2 - \dot{H})\rho f_Q$$
$$-\frac{3}{2}(\dot{H} + 3H^2)pf_Q - \frac{3}{2}H\partial_t[(\rho - p)f_Q]], \tag{10}$$
$$p_{eff} = \frac{-1}{f_Q\mathcal{L}_m - f_R}[p + (p + \mathcal{L}_m)f_T - \frac{1}{2}(f - Rf_R) - \frac{1}{2}(\dot{H} + 3H^2)(p - \rho)f_Q$$
$$+2H\partial_t f_R - \frac{1}{2}\partial_{tt}[(p - \rho)f_Q - f_R] + 2H\partial_t[(\rho + p)f_Q]], \tag{11}$$

where $H = \dot{a}a^{-1},\ R = -6\dot{H} - 12H^2$ and upper dot marks time derivative. Here, we ignored those terms which involved the second derivative of L_m w.r.t $g_{\mu\nu}$. In the case of perfect fluid L_m can either be "$\mathcal{L}_m = \rho$" or "$\mathcal{L}_m = -p$".

3. $f(R,T,Q) = R(1 + \alpha Q)$ Gravity

We intend to dicuss the cosmic evolution using matter conservation equation of more generic modified theory. Here, we will set $\mathcal{L}_m = \rho$ and we will take the simplest model "$f(R,T,Q) = R(1 + \alpha Q)''$ where α being the coupling parameter. In this model, the choice of $\alpha = 0$, results in Eisntein's formalism of GR.

For a flat FLRW universe, the non-zero components of FLRW equation for $p_{eff} = p + p_{DE}$ and $\rho_{eff} = \rho + \rho_{DE}$ are

$$3H^2 = \rho_{eff},$$
$$-2\dot{H} - 3H^2 = P_{eff}, \tag{12}$$

where dots being time derivative and components of ρ_{DE} and p_{DE} are given as follows

$$\rho_{DE}=\frac{-1}{1+3\alpha(p+\rho)(3H^2+\dot{H})}[3\alpha(-18H^4(p+\rho)+3H^2(\rho(p+\rho)+(5p-7\rho)$$
$$\dot{H})+\dot{H}(\rho(p+\rho)+3(-p+\rho)\dot{H})+3H^3(5\dot{p}-3\dot{\rho})+6H(\dot{H}(\dot{p}-\dot{\rho})+(p-\rho)\ddot{H}))],$$
$$p_{DE}=\frac{-1}{1+3\alpha(p+\rho)(3H^2+\dot{H})}[3\alpha(6H^4(-p+\rho)+2H^3(\dot{p}+5\dot{\rho})+4(-\dot{p}+\dot{\rho})\ddot{H}$$
$$+2H(9\dot{H}(-\dot{p}+\dot{\rho})+2(-2p+3\rho)\ddot{H})+\dot{H}(p(p+\rho)+(-11p+7\rho)\dot{H}-2\ddot{p}+2\ddot{\rho})+$$
$$H^2(3p(p+\rho)-(p-25\rho)\dot{H}-5\ddot{p}+3\ddot{\rho})+2(-p+\rho)H^3)],\tag{13}$$

and effective EoS ω_{eff} is

$$\omega_{eff}=[\rho+9\alpha(6H^4(p+\rho)+H^2(-5p+7\rho)\dot{H}+(p-\rho)\dot{H}^2)+H^3(-5\dot{p}+3\dot{\rho})+$$
$$2H(\dot{H}(-\dot{p}+\dot{\rho})+(-p+\rho)\ddot{H})]^{-1}[-3\alpha(6H^4\rho+2H^3(\dot{p}+5\dot{\rho})+4(-\dot{p}+\dot{\rho})\ddot{H}+6H$$
$$(3\dot{H}(-\dot{p}+\dot{\rho})+2\rho\ddot{H})+\dot{H}(7\rho\dot{H}-2\ddot{p}+2\ddot{\rho})+H^2(25\rho\dot{H}-5\ddot{p}+3\ddot{\rho})+2\rho H^3)$$
$$+p(1+3\alpha(6H^4+H^2\dot{H}+11\dot{H}^2+8H\ddot{H}+2H^3))].\tag{14}$$

The EoS of DE is, $\omega_{DE}=\frac{p_{DE}}{\rho_{DE}}$

$$\omega_{DE}=[6H^4(-p+\rho)+2H^3(\dot{p}+5\dot{\rho})+4(-\dot{p}+\dot{\rho})\ddot{H}+2H(9\dot{H}(-\dot{p}+\dot{\rho})+2(-2p+$$
$$3\rho)\ddot{H})+\dot{H}(p(p+\rho)+(-11p+7\rho)\dot{H}-2\ddot{p}+2\ddot{\rho})H^2(3p(p+\rho)-(p-25\rho)\dot{H}-5\ddot{p}$$
$$+3\ddot{\rho})+2(-p+\rho)H^3][-18H^4(p+\rho)+3H^2(\rho(p+\rho)+(5p-7\rho)\dot{H})+\dot{H}(\rho(p+\rho)$$
$$+3(-p+\rho)\dot{H})+3H^3(5\dot{p}-3\dot{\rho})+6H(\dot{H}(\dot{p}-\dot{\rho})+(p-\rho)\ddot{H})]^{-1}.\tag{15}$$

and conservation equation (7) takes the form

$$\dot{\rho}+3H(p+\rho)=\frac{9\alpha(2H^2+\dot{H})(3H^2+\dot{H})(2H(p+\rho)+\dot{p}+\dot{\rho})}{1+18\alpha(2H^2+\dot{H})^2}.\tag{16}$$

The revolutionary field equation $G_{\mu\nu}=8\pi GT_{\mu\nu}$ shows the connectedness of matter content of cosmos with geometry of the fabric of space-time, represented in GR. The LHS of the previously stated field equation show the Einstein tensor, which satisfy the Bianchi identities $\nabla_\nu G^\nu_\mu\equiv 0$ and RHS shows the EMT. If the covariance derivative of EMT is zero ($\nabla_\mu T^\nu_\mu=0$) then it shows the conservation of matter in every part of the universe. EFE can be explored on different choices of metric $g_{\mu\nu}$ and EMT $T_{\mu\nu}$. Although matter and geometry are on same footing but GR does not allow us to check the possible effects of nonminimal coupling between them. These limitations of GR vanished in recently developed theories like $f(R,T)$ and $f(R,T,Q)$ theories. In these theories EMT is not conserved ($\nabla_\mu T^\nu_\mu\neq 0$), we use this result to find the value of energy density. Such formation of energy density from the nonconserved EMT helps to study the role of non-minimal coupling in cosmic expansion. Before finding the value of $\rho(z)$ we should know the relation of $H(z)$. But here we will take the power law expansion in terms of red shift given as $H(z)=H_0(1+z)^{\frac{1}{m}}$, where m is the power law exponent.

Power law cosmology appears as a effective phenomenological explanation of the evolution of cosmos, it can describe the cosmic history including radiation phase, the phase of dark matter and the accelerating DE dominated epoch. Moreover, power law solutions renders evolution based on scale factor of the form as dust matter case ($m = 2/3$) or radiation dominated eras ($m = 1/2$). Also, $m \gtrsim 1$ predicts a late-time accelerating cosmos. It provides an interesting alternative to deal with the problems like (age, flatness and horizon problems) linked with the standard model. Evolution of power law model has been discussed in various articles[35], for instance it addresses the horizon, flatness and age problems for the parametric value $m \geq 1$[36]. These type of solutions are found to be consistent with various data sets including nucleosynthesis[37,38], with the age of high-redshift objects such as globular clusters[37,38], with the SNIa data[39], and with X-ray gas mass fraction measurements of galaxy clusters[40]. Applying ower law cosmology, authors have discussed the angular size-redshift data of compact radio sources[41], the gravitational lensing statistics and SNIa magnitude-redshift relation[38,42].

In this scenario, by using the equation (16) and converting it into redshift by using the relation $a(t) = \frac{1}{1+z}$ and $\frac{d}{dt} = -(1+z)H\frac{d}{dz}$ whereas $p = p(z)$. Energy density is found as

$$\rho(z) = e^{(1+\omega)(3\log(1+z) - \frac{m(-1+3m)(1+3\omega)\log[-m^2+9H_0^4(-1+2m)(1+z)^{\frac{4}{m}}\alpha(1-\omega+m(-1+3\omega))]}{4-4\omega+4m(-1+3\omega)})}c \tag{17}$$

where c is integration constant. As energy density is found to be in an exponential form so it will remain positive for all values of unknowns parameters like α, ω, m, z. It will only depend on constant of integration c when we take negative value of c then energy density will be negative or less than zero otherwise for all positive values of c energy density will remain positive.

One can also get the relation between time and redshift as

$$t = \left(\frac{1}{1+z}\right)^{\frac{1}{m}}. \tag{18}$$

Using the value of $\rho(z)$, one can get ρ_{eff} and p_{eff} in terms of redshift as and we take $c = 10$ and $\omega = 1$

$$\begin{aligned}\rho_{eff} = &-[10(1+z)^6(m^2 + 180H_0^4m^2(1+z)^{\frac{4}{m}}\alpha - 972H_0^8(-1+2m)(1+z)^{\frac{8}{m}}\alpha^2)] \\ &[m(-m+18H_0^4(-1+2m)(1+z)^{\frac{4}{m}}\alpha)^{3m} + 60H_0^2(1+z)^{6+\frac{2}{m}}\alpha(m-3m^2+ \\ &18H_0^4(1+m(-5+6m))(1+z)^{\frac{4}{m}}\alpha]^{-1},\end{aligned} \tag{19}$$

$$\begin{aligned}p_{eff} = &[10(1+z)^6(m^4 + 12H_0^4m^2(-1+6m(2+5m))(1+z)^{\frac{4}{m}}\alpha + 108H_0^8m(-1+ \\ &2m)(-16-51m+78m^2)(1+z)^{\frac{8}{m}}\alpha^2 - 34992H_0^{12}(1-2m)^2(-1+m)(1+z)^{\frac{12}{m}}\alpha^3)] \\ &[m(m-18H_0^4(-1+2m)(1+z)^{\frac{4}{m}}\alpha)^2(60H_0^2(-1+3m)(1+z)^{6+\frac{2}{m}}\alpha + m(m(-m \\ &+18H_0^4(-1+2m)(1+z)^{\frac{4}{m}}\alpha))^{-1+3m})]^{-1},\end{aligned} \tag{20}$$

and effective EoS in term of redshift can be written as

$$\omega_{eff} = -2 + \frac{2}{m} - \frac{4m}{-m + 18H_0^4(-1+2m)(1+z)^{\frac{4}{m}}\alpha} - [m(2+m) + 6H_0^4(2+9m$$
$$(3+2m))(1+z)^{\frac{4}{m}}\alpha][m^2 + 180H_0^4 m^2(1+z)^{\frac{4}{m}}\alpha - 972H_0^8(-1+2m)(1+z)^{\frac{8}{m}}\alpha^2]^{-1}.$$

Cosmic acceleration can be measured through a dimensionless cosmological function known as the deceleration parameter q. Here, q is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{1}{m} - 1 \tag{21}$$

q characterizes the accelerating or decelerating behavior of cosmos, here, $q < 0$ explains an accelerating epoch, whereas $q > 0$ describes decelerating epoch. In power law cosmology we require $m > 0$ to restrict q as $q > -1$. Graphical representation of effective components ρ_{eff}, EoS ω_{eff} are shown in Fig. 2. In this discussion, we choose the following values of unknown parameters $\alpha = 10$, and $m = 2$. For this value of m, deceleration parameter is -0.5 which favors the expanding behavior of cosmos. We set the parameters in a way to keep the positivity of ρ_{eff}. It can be seen that ρ_{eff} is positive and increasing function as shown on right plot and ω_{eff} and ω_{DE} are exactly equal to -1, shown in FIG. 1., representing the ΛCDM epoch in accordance with recent observations from Plank's data[2].

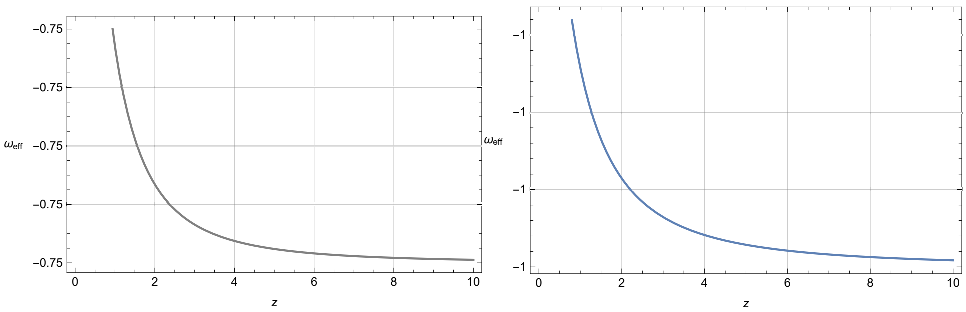

Fig. 1. Left part of the Figure represents the evolution of ω_{eff} for $m = 1.6$ whereas right side shows the behavior ω_{eff} for $m = 2$. Parameters are chosen as $\alpha = 10$ and $H_0 = 67.3$[2].

4. Energy conditions

Now, we will discus the energy conditions for our particular model of $f(R,T,Q)$ gravity which is $R(1+\alpha Q)$ by considering FLRW metric.

The validity of above mentioned inequalities is totaly model dependent. In this model these depend on two parameters "α" coupling parameter, and "m" the power law exponent. Here, we fixed $H_0 = 67.3$, $c = 10$. $\rho_{eff} \geq 0$ is valid for all positive values of α, i.e., $\alpha > 0$ and $m > 1$. $\rho_{eff} + peff \geq 0$ is valid for $\alpha > 0$, and m is restrict in this model between $1 < m < 2$, this inequality is not valid for greater

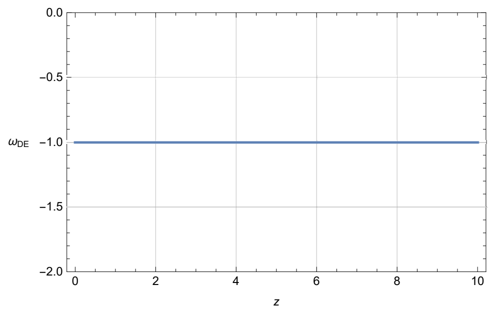

Fig. 2. Evolution of ω_{DE}. Herein, we set $\alpha = 10$, $m = 2$, and $H_0 = 67.3^2$.

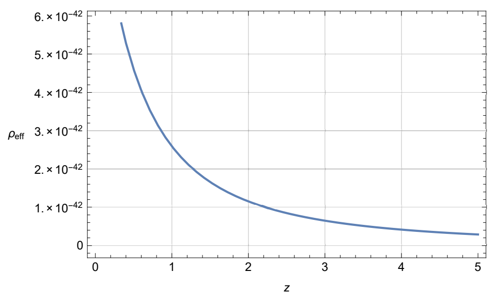

Fig. 3. The behavior of ρ_{eff} versus z. Herein, we set $\alpha = 10$, $m = 2$, and $H_0 = 67.3^2$.

values of m. NEC is also valid for the positive values of α and m should be between $1 < m < 2$. We showed the valid regions of NEC and WEC in FIG. 3. and FIG. 4. respectively.

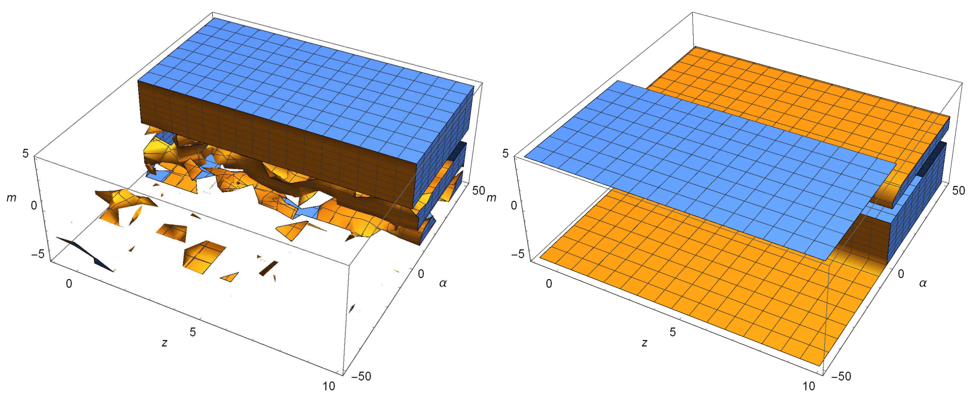

Fig. 4. Left part of Figure depicts the validity region for $\rho_{eff} \geq 0$ whereas right side shows the validity region for $\rho_{eff} + p_{eff} \geq 0$ versus z.

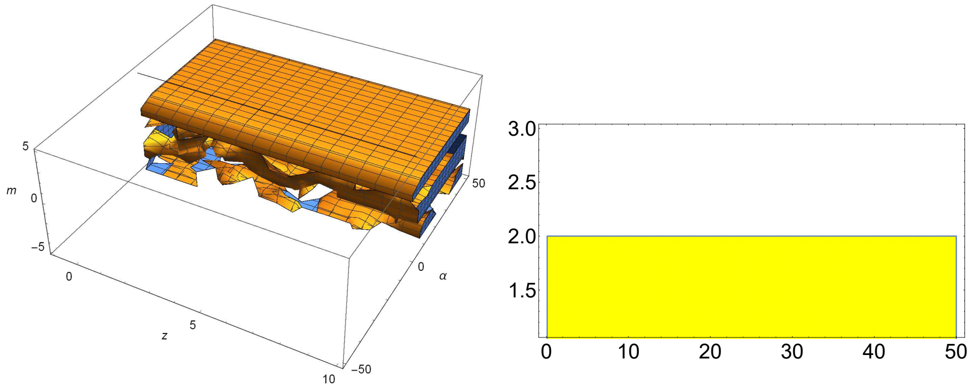

Fig. 5. Figure on the left represents the validity region for WEC ($\rho_{eff} \geq 0$, $\rho_{eff} + p_{eff} \geq 0$) in $3D$ whereas the figure on the right side also shows the validity region for WEC in $2D$. Herein, we set $H_0 = 67.3$, versus z, and for $2D$ plot we set $z = 0$.

5. Discussion

In this manuscript, we have developed a cosmological scenario from the complicated non-minimal coupling of matter and geometry. We consider a simplest case of non-minimal coupling in the $f(R, T, Q)$ modified theory in the form of model $f(R, T, Q) = R(1 + \alpha Q)$. Dynamical equations are presented in section **III**, where we consider the power law cosmology to find a relation for energy density ρ. Using Eq.(17), it is obvious to find the expressions of effective energy momentum tensor and its components. In power law cosmology, one can represent the cosmic history depending on the choice of parameter m. Here, we set parameter m according to the evolution of q as per recent observational data. It is to be noted that we set the choice of parameters α and m as per validity ranges expressed in Table **1**, where we develop the constraints on coupling parameter α for different values of m satisfying WEC and NEC. It is found that energy constraints are valid for positive values of α. For the discussion on evolution of cosmos and behavior of ρ_{eff} and ω_{eff}, we choose $\alpha = 10$. In Fig. 1, we set $m = 2$ with $q = -0.5$ to see the evolution of ω_{eff}, it is found that WEC is satisfied and $\omega_{eff} \to -1$ validating the current cosmic epoch [2]. In Fig. 1, we show the behavior of ω_{DE} which approaches to phantom divide line. Evolution of WEC and NEC versus redshift z is presented in Fig. 3-5.

In literature, observational constraints have been developed on the choice of power law exponent m, cosmological parameters q and H_0. Kaeonikhom et al. [49] explored the phantom power law cosmology using cosmological observations from CMB, BAO and observational Hubble data, they found the best fit value of power law exponent as $m \approx -6.51^{+0.24}_{-0.25}$. In [48], Kumar found the constraints on Hubble and deceleration parameters from the latest $H(z)$ and SNeIa data as $q = -0.18^{+0.12}_{-0.12}$, $H_0 = 68.43^{+2.84}_{-2.80}$kms-1Mpc-1 and $q = -0.38^{+0.05}_{-0.05}$, $H_0 = 69.18^{+0.55}_{-0.54}$ kms-1Mpc-1 respectively. The combination of $H(z)$ and SNe Ia data yields the constraints $q = -0.34^{+0.05}_{-0.05}$, $H_0 = 69.18^{+0.55}_{-0.54}$kms-1Mpc-1. The consistent observational constraints on both of the parameters q and H_0 according to latest 28 points of $H(z)$ are found as $q = -0.0451^{+0.0.0614}_{-0.0625}$, $H_0 = 65.2299^{+2.4862}_{-2.4607}$, in case of Union2.1 SN data, these parameters take the values $q = -0.3077^{+0.1045}_{-0.1036}$, $H_0 = 68.7702^{+1.4052}_{-1.3754}$ [50]. Using the data set of Kumar[48] and Rani et al.[50], we choose the parameter m and develop the ranges of ω_{eff} as shown in Table **2**. For $m = 2$, ω_{eff} is found to be -0.1 which agrees with the observational results of Planck+WMAP+H_0[2]. Also, for the choice of $m = 1.4445$, results of ω_{eff} are consistent with the observational data of WMAP9[51]. It is found that non-minimal coupling provides more degrees of freedom to explore various issues of cosmos. A more detailed study on cosmic evolution for more generic model in this coupling theory is carried out in[?] .

Data	q	H_0	m	ω_{eff}
$H(z)$ (14 points)[48]	$-0.18^{+0.12}_{-0.12}$	$68.43^{+2.84}_{-2.80}$	1.221	-0.362
SN (Union2)[48]	$-0.38^{+0.05}_{-0.05}$	$69.18^{+0.55}_{-0.54}$	1.613	-0.760
$H(z) + SN(Union2)$[48]	$-0.34^{+0.05}_{-0.05}$	$68.93^{+0.53}_{-0.52}$	1.516	-0.681
$H(z)$ (29 points)[50]	$-0.0451^{+0.0614}_{-0.0625}$	$65.2299^{+2.4862}_{-2.4607}$	1.0473	-0.090
SN (Union2.1)[50]	$-0.3077^{+0.1045}_{-0.1036}$	$68.7702^{+1.4052}_{-1.3754}$	1.4445	-0.615

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