Relativistic stars in degenerate higher-order scalar-tensor theories after GW170817

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We study relativistic stars in degenerate higher-order scalar-tensor theories that evade the constraint on the speed of gravitational waves imposed by GW170817. It is shown that the exterior metric is given by the usual Schwarzschild solution if the lower order Horndeski terms are ignored in the Lagrangian and a shift symmetry is assumed. However, this class of theories exhibits partial breaking of Vainshtein screening in the stellar interior and thus modifies the structure of a star. Employing a simple concrete model, we show that for high-density stars the mass-radius relation is altered significantly even if the parameters are chosen so that only a tiny correction is expected in the Newtonian regime. We also find that, depending on the parameters, there is a maximum central density above which solutions cease to exist. **See[1] for more details.**

Keywords: Modified Gravity; Horndeski Theory

1. Introduction

The nearly simultaneous detection of gravitational waves GW170817 and the γ-ray burst GRB 170817A places a very tight constraint on the speed of gravitational waves, $c_{\rm GW}$. The deviation of $c_{\rm GW}$ from the speed of light is less than 1 part in 10^{15}. This can be translated to constraints on modified gravity such as scalar-tensor theories, vector-tensor theories, massive gravity, and Hořava gravity. In particular, in the context of the Horndeski theory (the most general scalar-tensor theory having second-order equations of motion, two of the four free functions in the action are strongly constrained, leaving only the simple, traditional form of nonminimal coupling of the scalar degree of freedom to the Ricci scalar, *i.e.,* the "$f(\phi)\mathcal{R}$"-type coupling.

However, it has been pointed out that there still remains an interesting, nontrivial class of scalar-tensor theories beyond Horndeski that can evade the gravitational wave constraint as well as solar-system tests. Such theories have higher-order equations of motion as they are more general than the Horndeski theory, but the system is degenerate and hence is free from the dangerous extra degree of freedom that causes Ostrogradski instability. They are called degenerate higher-order scalar-tensor (DHOST) theory.

In this paper, we consider relativistic stars in DHOST theories that are more general than the GLPV theory but evade the constraint on the speed of gravitational waves.

2. Field equations

The action of the quadratic DHOST theory we study is given by

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[f(X)\mathcal{R} + \sum_{I=1}^{5}\mathcal{L}_I + \mathcal{L}_\mathrm{m}\right], \tag{1}$$

where $\mathcal{R}$ is the Ricci scalar, $X := \phi_\mu\phi^\mu$, and

$$\begin{aligned}&\mathcal{L}_1 := A_1(X)\phi_{\mu\nu}\phi^{\mu\nu}, \quad \mathcal{L}_2 := A_2(X)(\Box\phi)^2, \quad \mathcal{L}_3 := A_3(X)\Box\phi\phi^\mu\phi_{\mu\nu}\phi^\nu,\\ &\mathcal{L}_4 := A_4(X)\phi^\mu\phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu, \quad \mathcal{L}_5 := A_5(X)(\phi^\mu\phi_{\mu\nu}\phi^\nu)^2,\end{aligned} \tag{2}$$

with $\phi_\mu = \nabla_\mu\phi$ and $\phi_{\mu\nu} = \nabla_\mu\nabla_\nu\phi$. The functions $A_I(X)$ must be subject to certain conditions in order for the theory to be degenerate and satisfy $c_\mathrm{GW} = 1$, as explained shortly.

We require that the speed of gravitational waves, c_GW, is equal to the speed of light. In our theory c_GW is given by $c_\mathrm{GW}^2 = f/(f - XA_1)$, so that

$$A_1 = 0. \tag{3}$$

The degeneracy conditions read $A_2 = -A_1 = 0$ and

$$A_4 = -\frac{1}{8f}\left(8A_3f - 48f_X^2 - 8A_3f_XX + A_3^2X^2\right), \tag{4}$$

$$A_5 = \frac{A_3}{2f}\left(4f_X + A_3X\right). \tag{5}$$

We thus have two free functions, A_3 and f, in the quadratic DHOST sector with $c_\mathrm{GW} = 1$. Following[2], we introduce $B_1 := (X/4f)(4f_X + XA_3)$, and use this instead of A_3.

We consider a static and spherically symmetric metric,

$$\mathrm{d}s^2 = -e^{\nu(r)}\mathrm{d}t^2 + e^{\lambda(r)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2. \tag{6}$$

The scalar field is taken to be

$$\phi(t, r) = vt + \psi(r), \tag{7}$$

where $v\,(\neq 0)$ is a constant. Even though ϕ is linearly dependent on the time coordinate, it is consistent with the static spacetime because the action (1) possesses a shift symmetry, $\phi \to \phi + c$, and ϕ without derivatives does not appear in the field equations.

The field equations are given by

$$\mathcal{E}_\mu^\nu = T_\mu^\nu, \quad \nabla_\mu J^\mu = 0, \tag{8}$$

where $\mathcal{E}_{\mu\nu}$ is obtained by varying the action with respect to the metric and J^μ is the shift current defined by $\sqrt{-g}J^\mu = \delta S/\delta\phi_\mu$. The energy-momentum tensor is of the form $T_\mu^\nu = \mathrm{diag}(-\rho, P, P, P)$. The radial component of the conservation equations, $\nabla_\mu T_\nu^\mu = 0$, reads

$$P' = -\frac{\nu'}{2}(\rho + P), \tag{9}$$

where $' := \mathrm{d}/\mathrm{d}r$.

With direct calculation we find that $J^r \propto \mathcal{E}_{tr}$. Therefore, the gravitational field equation $\mathcal{E}_{tr} = 0$ requires that J^r vanishes. Then, the field equation for the scalar field is automatically satisfied.

To write the field equations explicitly, it is more convenient to use $X = -e^{-\nu}v^2 + e^{-\lambda}\psi'^2$ instead of ψ. In terms of X, we have the equations of the form

$$\mathcal{E}_t^t = b_1\nu'' + b_2 X'' + \widetilde{\mathcal{E}}_t(\nu, \nu', \lambda, \lambda', X, X'), \tag{10}$$

$$\mathcal{E}_r^r = c_1\nu'' + c_2 X'' + \widetilde{\mathcal{E}}_r(\nu, \nu', \lambda, \lambda', X, X'), \tag{11}$$

$$\psi' J^r = c_1\nu'' + c_2 X'' + \widetilde{\mathcal{E}}_J(\nu, \nu', \lambda, \lambda', X, X'). \tag{12}$$

We see that $\mathcal{E}_r^r$ and $\psi' J^r$ have the same coefficients c_1 and c_2. Moreover, we find by an explicit computation that $\mathcal{E}_r^r$ and $\psi' J^r$ are linearly dependent on λ' and their coefficients are also the same. Therefore, by taking the combination $\mathcal{E}_r^r - \psi' J^r$ one can remove ν'', X'', and λ'. Then, the field equation $\mathcal{E}_r^r - \psi' J^r = P$ can be solved for λ to give

$$e^{\lambda} = \mathcal{F}_\lambda(\nu, \nu', X, X', P), \tag{13}$$

where the explicit expresson for $\mathcal{F}_\lambda$ is not important for the moment.

Using Eq. (13), we can eliminate λ and λ' from Eqs. (10) and (12). In doing so we replace P' with ν', ρ, and P by using Eq. (9). We then obtain

$$\psi' J^r = k_1\nu'' + k_2 X'' + \mathcal{J}_1(\nu, \nu', X, X', \rho, P) = 0, \tag{14}$$

where $k_{1,2} = k_{1,2}(\nu, \nu', X, X', P)$. The field equation $\mathcal{E}_t^t + \rho = 0$ can also be written in the form

$$k_1\nu'' + k_2 X'' + \mathcal{J}_2(\nu, \nu', X, X', \rho, P) = 0. \tag{15}$$

Note that we have the same coefficients k_1 and k_2. This is due to the degeneracy conditions. We thus arrive at a first-order equation, $\mathcal{J}_1 = \mathcal{J}_2$, which can be solved for X' as

$$X' = \mathcal{F}_1(\nu, X, \rho, P)\nu' + \frac{\mathcal{F}_2(\nu, X, \rho, P)}{r}, \tag{16}$$

where $\mathcal{F}_1$ and $\mathcal{F}_2$ are complicated. Their explicit form is presented in the Appendix. Finally, we use Eq. (16) to eliminate X' and X'' from Eq. (14). This manipulation also removes ν'', as it should be because the theory is degenerate. We thus arrive at

$$\nu' = \mathcal{F}_3(\nu, X, \rho, \rho', P), \tag{17}$$

where the explicit expression of $\mathcal{F}_3$ is extremely long and we do not present it here.

We have thus obtained our basic equations describing the Tolman-Oppenheimer-Volkoff system in DHOST theories. Given the equation of state relating ρ and P, one can integrate Eqs. (9), (16), and (17) to determine $P = P(r)$, $\nu = \nu(r)$, and $X = X(r)$. Equation (13) can then be used to obtain $\lambda = \lambda(r)$.

3. Numerical Results

Having thus obtained the equations describing the TOV system in DHOST theories, we solve them numerically. As a specific example, we study the model of[2] which is characterized by two parameters. The equation of state we use is given by

$$\rho = \left(\frac{P}{K}\right)^{1/2} + P, \quad K = 123 M_\odot. \tag{18}$$

A example of our numerical results is shown in Fig. 1. We take the model parameters such that only tiny corrections arise in the Newtonian regime. Nevertheless, we find large deviation from the result of general relativity in the mass-radius relation of relativistic stars.

More detailed description is find in[1].

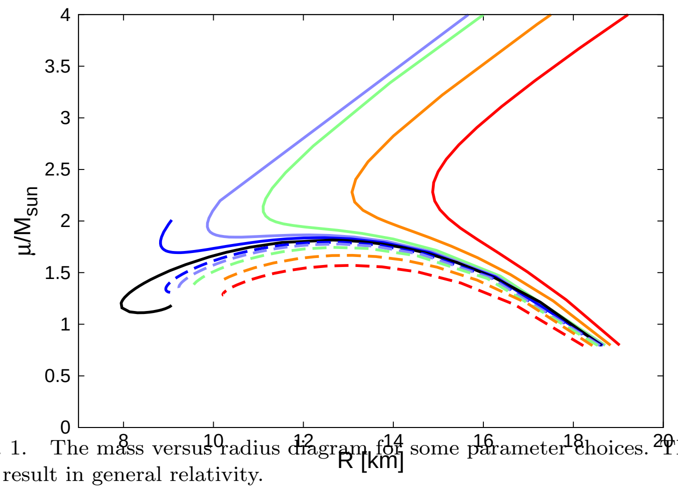

Fig. 1. The mass versus radius diagram for some parameter choices. The black curve represents the result in general relativity.

References

1. T. Kobayashi and T. Hiramatsu, Phys. Rev. D **97**, no. 10, 104012 (2018) [arXiv:1803.10510 [gr-qc]].
2. M. Crisostomi and K. Koyama, Phys. Rev. D **97**, no. 8, 084004 (2018). [arXiv:1712.06556 [astro-ph.CO]].