The electromagnetic interaction in the Hořava-Lifshitz gravity

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We analyse the electromagnetic-gravity interaction in a pure Hořava-Lifshitz framework. To do so we formulate the Hořava-Lifshitz gravity in $4+1$ dimensions and perform a Kaluza-Klein (KK) reduction to 3+1 dimensions. The critical values of the dimensionless coupling constant in the kinetic term of the action are the relativistic value $\lambda = 1$, and $\lambda = 1/4$. The relativistic symmetry of the kinetic term for $\lambda = 1$ is broken by the potential terms in the Horava-Lifshitz formulation leaving the only critical value of the action to be $\lambda = 1/4$. It is the kinetic conformal point for the nonrelativistic electromagnetic-gravity interaction. In distinction, the corresponding kinetic conformal value for pure Hořava-Lifshitz gravity in $3+1$ dimensions is $\lambda = 1/3$. We analyse the geometrical structure of the critical and noncritical cases, they correspond to different theories. The physical degrees of freedom propagated by the theory for $\lambda = 1/4$, when the KK scalar is taken in its ground state as is usually done, are exactly the ones corresponding to a graviton and a photon, without any additional scalar field. The field equations for the gauge vector have exactly the same form as the electromagnetic equations coupled to gravity in General Relativity, they are now coupled to non-relativistic Hořava-Lifshitz gravity. The transverse traceless degrees of freedom of the graviton and the transverse degrees of freedom of the gauge vector propagate with the same velocity which can be taken to be the speed of light.

Keywords: Hořava gravity, quantum gravity.

1. Introduction

Hořva-Lifshitz gravity[1,2] is a candidate for a perturbative renormalizable quantum theory. It is a non-relativistic theory based on a anisotropic scaling of time and space. In distinction to what occurs in general relativity (GR), for a particular scaling $z = D$ in $D+1$ dimensions the kinetic coupling constant becomes dimensionless and consequently a candidate for a renormalizable theory[3–5]. One important goal in this anisotropic framework is to introduce the gravity + matter interaction. In this work we approach this point by considering a non-proyectable Horăva-Lifshitz gravity in $4+1$ dimensions with a $z = 4$ scaling and then reducing the theory *a la* Kaluza-Klein, to $3+1$ diemnsions. In this way we will obtain the electromagnetic + graviry coupling in a manifestly anisotropic formulation.

2. Hořava-Lifshitz Gravity in Five Dimensions

We consider the non-proyectable Hořava-Lifshitz action in a foliated $4+1$ dimensional Lorentzian manifold. The 4-dimensional slices are Riemannian manifolds with metric $G_{\mu\nu}dx^{\mu}\otimes dx^{\nu}$. The action is given by

$$S\left(G_{\mu\nu},N_{\rho},N\right)=\int dtdx^{4}N\sqrt{G}\Big[K_{\mu\nu}K^{\mu\nu}-\lambda K^{2}+\beta^{(4)}R+\alpha a_{\mu}a^{\mu} +V\left(G_{\mu\nu},N\right)\Big], \quad (1)$$

where $K_{\mu\nu}$ is the extrinsic curvature

$$K_{\mu\nu}=\frac{1}{2N}\left(\dot{G}_{\mu\nu}-\nabla_{\mu}N_{\nu}-\nabla_{\nu}N_{\mu}\right), \quad (2)$$

with the trace

$$K=G^{\mu\nu}K_{\mu\nu}, \quad (3)$$

λ is a dimensionless coupling constant, α and β are the remaining couplings when only the low energy interaction terms are left. The potential of the theory is

$$\beta^{(4)}R+\alpha a_{\mu}a^{\mu}+V\left(G_{\mu\nu},N\right), \quad (4)$$

where $V\left(G_{\mu\nu},N\right)$ includes the interaction terms of order beyond 2, in space like derivatives. In $4+1$ dimensions the highest order terms corresponding to the $z=4$ scaling contain all terms constructed in terms of the 4-dimensional Riemannian tensor and covariant derivatives of the lapse N up to 8 space like derivatives. a_{μ} denotes the logarithmic derivative of N.

In this work we start the analysis by considering the lowest order terms, that is the low energy ones and we disregard the term $V\left(G_{\mu\nu},N\right)$. The Hamiltonian formulation of the action can be obtained following the Legendre transformation. The conjugate momentum to $G_{\mu\nu}$ are given by

$$\pi^{\mu\nu}=\frac{\partial\mathcal{L}}{\partial\dot{G}_{\mu\nu}}=\sqrt{G}\left(K^{\mu\nu}-\lambda G^{\mu\nu}K\right), \quad (5)$$

with trace

$$\pi=G_{\mu\nu}\pi^{\mu\nu}=\sqrt{G}\left(1-4\lambda\right)K. \quad (6)$$

At this stage we have to distinguish two different theories, one corresponding to $\lambda=1/4$ and the other to $\lambda\neq1/4$. The propagating physical degrees of freedom are different. The corresponding splitting of pure Hořava-Lifshitz gravity in $3+1$ dimensions correspond to $\lambda=1/3$ and $\lambda\neq1/3$. In the case $\lambda=1/3$ the theory propagates exactly the same degrees of freedom as GR while in the $\lambda=1/4$ case in $3+1$ dimensions, after the KK reduction, leaving constant the KK scalar dilaton field in the action the theory propagates $2+2$ degrees of freedom corresponding to the transverse electromagnetic mode just as in GR coupled to the energy momentum tensor $T^{\mu\nu}$ of the electromagnetic interaction. The propagation of the gravity and electromagnetic degrees of freedom occurs with the same speed expressed in terms of the coupling constant β. Moreover, the field equations of the theory arising from variations of the action with respect to the electromagnetic potential are exactly as in the GR + $T^{\mu\nu}$ theory. The corresponding first class constraint is also as in that theory. The gauge symmetry of the electromagnetic interaction arises from the former diffeomorphism invariance in the 4 + 1 theory.

The Hamiltonian corresponding to $\lambda \neq 1/4$ case in 4 + 1 is given by

$$\mathcal{H} = \sqrt{G}N\left[\frac{\pi^{\mu\nu}\pi_{\mu\nu}}{G} + \frac{\lambda}{(1-4\lambda)}\frac{\pi^2}{G} - \beta^{(4)}R - \alpha a_\mu a^\mu\right] + 2\pi^{\mu\nu}\nabla_\mu N_\nu. \tag{7}$$

The constraint arising from variations of the lapse N is given by

$$\frac{\pi^{\mu\nu}\pi_{\mu\nu}}{G} + \frac{\lambda}{(1-4\lambda)}\frac{\pi^2}{G} - \beta^{(4)}R + \alpha a_\mu a^\mu + 2\alpha\nabla_\mu a^\mu = 0. \tag{8}$$

It turn out to be a second class constraint. In order to reduce to the 3 + 1 theory we perform the following canonical transformation

$$G_{\mu\nu} = \begin{pmatrix} \gamma_{ij} + \phi A_i A_j & \phi A_j \\ \phi A_i & \phi \end{pmatrix},$$

and

$$p^{ij} \equiv \pi^{ij} \tag{9}$$

$$p^i \equiv 2\phi A_j \pi^{ij} + 2\phi\pi^{4i} \tag{10}$$

$$p \equiv A_i A_j \pi^{ij} + 2A_i\pi^{4i} + \pi^{44}, \tag{11}$$

where p^{ij}, p^i, p are the conjugate momenta to γ_{ij} the 3-dimensional metric, A_i the gauge vector and ϕ the scalar dilaton field.

The canonical action becomes

$$S = \int d^3x dt \Bigg\{ p^{ij}\dot{\gamma}_{ij} + p\dot{\phi} + \frac{1}{2}p^i\dot{A}_i - \frac{N}{\sqrt{\gamma\phi}}\Bigg[\phi^2p^2 + p^{ij}p_{ij} + \frac{1}{2\phi}p^ip_i$$
$$+ \frac{\lambda}{(1-4\lambda)}\left(\phi^2p^2 +^{(3)}\pi^2 + 2\phi^{(3)}\pi p\right) - \gamma\phi\beta^{(4)}R - \gamma\phi\alpha a_i a^i\Bigg] \tag{12}$$
$$+ N_iH^i + N_4H^4\Bigg\},$$

where

$$^{(4)}R =^{(3)} R - \frac{1}{4}\phi F^{ij}F_{ij} - \frac{2}{\sqrt{\phi}}\nabla_i\nabla^i\sqrt{\phi}. \tag{13}$$

This theory propagates 6 physical degrees of freedom 2 of them corresponding to the graviton field, 2 to the electromagnetic gauge field and two scalar fields.

A perturbative analysis shows that the corresponding propagating wave equations for the physical degrees of freedom are

$$\ddot{h}^{TT}_{ij} = \beta\Delta h^{TT}_{ij} \tag{14}$$

$$\ddot{A}^T_i = \beta\Delta A^T_i \tag{15}$$

$$2\ddot{\phi} - \ddot{h}^T = \beta\Delta\left(2\phi - h^T\right) \tag{16}$$

$$\ddot{\phi} + \ddot{h}^T = 2\beta \frac{(1-\lambda)}{(1-4\lambda)} \frac{(3\beta - 2\alpha)}{2\alpha} \Delta \left(\phi + h^T\right) \tag{17}$$

In the theory $\lambda = 1/4$ there are $2+2+1$ degrees of freedom with evolution equations given by (14), (15) and (16). Finally if in the $\lambda = 1/4$ action we assume that the dilaton field is in its ground state, there are only $2+2$ degrees of freedom with evolution equations given by (14) and (15). Both excitations propagates with the same speed $1/\sqrt{\beta}$.

References

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