SHORT WAVELENGTH SCATTERING IN A SCHWARSCHILD FIELD

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A complete classification of papers , books and preprints concerning scattering of waves and particles by Black Holes would be given by following parameters: a) metrics examined, b) spin (Field's spin scattered), c) particles: mass/massless, charged/uncharged, etc. d) approach used, particularly: d1) weak gravitational field approach d2) exact metrics d3) quantum gravitational approach d4) exact mathematical treatments, which contains rigorous but no final formula for scattering phases, amplitudes, or scattering(absorption) cross sections, see, e.g. [21] d5) applications for astrophysical situations. I can list here more about 300 papers, or books , which does not cover points d4) and d5), but treat gravitational scattering. There is need also to mention that significant progress has been made in inverse scattering problem, when the metrics is found for a given scattering cross section. Some important results has been obtained in 3 body classical and quantum scattering problem.

To limit to direct scattering problem and to Schwarzschild metrics (or simplest scatterers as: Newtonian field of a point mass) there is need to tell, that there are few approaches to the problem, when means quantum mechanics, but classic scattering theory begun with a very complete study by Y. Hagihara of orbits in gravitational fields (1931). Only one paper I shall cite here which was less observed, namely by Mielnik, B. and Plebansky, J. (1962). A gold result in this area of research is an exact absorption cross section for photons by an exact Reissner-Nordstrom metrics with an arbitrary electric charge , obtained by N.R. Sibgatullin [20], unfortunately died in 2003. The weak gravitational field approach and interacting quantum fields approach was reviewed almost exhaustively recently in the book [2]. c) The exact gravitational field approach and approximate scattered fields. This approach was developed for waves and particles (quantum mechanic) since 1959 by Sir Ch. Darwin(1959, 1961, [3]), K.W. Ford and J.A. Wheeler (1959, [5]), A.B. Gaina (1980-1989 [7]-[8], [11]-[12]), F.A. Handler, R. Matzner (1980-1988), Cecile DeWitt-Morette and coauthors (1984, 1992 [6]), Decaninni, et all (2011). One of first results obtained using this approach– the absence of absorption of scalar waves in a Schwarzschild gravitational field (Matzner, 1969) was so strange for Black Holes physicists, that Wheeler (1971) has soon shown that Black Holes absorb particles and waves and can form quasibound states (when $E \leq mc^2$) around black holes. All approaches mentioned above were adopted by many researchers and results generally agree, except for some very particular cases. In 1987 [7] I have shown with a pupil, that computing scattering matrix would allow to calculate automatically by investigating the poles of the S-matrix the spectrum of quasibound states for mass particles. Generalizations for Dirac fields and exact metrics were given by Gaina (1980, 1983), Doran and Lasenby (2001). I would no discuss here absorption of scalar (mass) particles and spin1/2 particles examined in [11-15].

General background of the scattering problem in Black Holes spacetimes

To compute the scattering cross section quantum mechanically in the short wavelength in a Schwarzschild field in a short wavelength limit for mass particles I have used 1) the variables separation in wave equations (Wave equation, Klein-Gordon, Dirac) and it is possible to generalize this for higher spins) 2) obtained analytical solutions to radial equations in the long wavelength limit or I am writing the scattering phase integral in the short wave limit by a JWKB, which could be in general complex due to absorption 3) I am matching solutions in the long wavelength limit and compute the scattering phase, or calculate the phase integral , which , in general is given by an elliptic integral or elliptic integrals. In some cases, I am showing bellow, a Born perturbation procedure could be developed to evaluate the scattering phase integrals to avoid calculations of elliptic integrals.

We start with a Schwarzshild metric, responding to a mass M of a Black hole:

$$ds^2 = \left(1 - \frac{2MG}{c^2 r}\right) dt^2 - \left(1 - \frac{2MG}{c^2 r}\right)^{-1} dr^2 - r^2(d\theta^2 + sin^2\theta d\varphi^2) \quad (1)$$

The events horizon of this metric is placed at $r = \frac{2GM}{c^2}$ (1.a)

We shall use below the system of units $c = \hbar = G = 1$ if not a opposite is evident.

The most important case when the wave equations and relativistic Quantum mechanical equations can be solved analytically is the case of long wavelength limit. In this case not only the massless Scalar, Electromagnetic and s=2 (gravitational) and mass Klein-Gordon wave equations could be solved analytically in terms of Coulombian wave functions and Heun's (or hypergeometric functions), but also mass Dirac equation and mass Proca equations could be solved analytically first in a Kerr-Newman spacetime, and second in a Schwarzschild spacetime be included here. Scattering and absorption cross sections could be obtained in all this cases.

This procedure (1980, 1988) gives good results for ultrarelativistic spinless particles (or spinless photons), while in a case of non-relativistic particles it gives an error to about 18, 75% [12]. But it is normally to expect, that spin effects would arise in a higher approximation if take the spinless case as the null approximation . This is just the case, when the scattering occur effectively by a parabolic barrier, while the region just close to the horizon, which could be important for spin ½ particles (or fermions in general) does not contribute essentially. Otherwise, a potential well could arise between the centripetal potential barrier and the horizon, which is not related with Newtonian –like (or combined Newtonian-like -Coulombian-like) potential well. Then the case of bosons and fermions should be examined with care, taking into account, that fermions does not support superradiation . Only in some space-time regions results for bosons and fermions (scattering phases, transmission coefficients) agree sufficiently well.

Bellow, I am developing a Born approximation procedure for the scattering phases calculation valid in the short wavelengths limit $\lambda \equiv c/\nu \ll R_G \equiv \frac{2GM}{c^2}$ all the speeds of the particles ($0 < \frac{v}{c} \leq 1$) and which gives satisfactory result for all the cases.

Let us consider the Klein –Gordon equation in the Schwarzschild metric (gravitational external background)[2].The Wave function of the Klein-Gordon equation could be represented in the form:

$$\Phi_{E,l,m=\frac{1}{r}R_{E,l}(r)Y_l^m(\theta)e^{)iEt}} \tag{2}$$

where $Y_l^m(\theta)$ – are the spherical harmonics. The scattering amplitude and the cross section can be computed with the exactitude of a constant phase angle using the scattering angle calculated before. In view of the calculation of the constant phase we must apply consequently the JWKB method . The radial part of the wave function $R_{E,l}(r)$ satisfies the Regge-Wheeler equation:

$$\frac{d^2R}{dr^{*2}} + (\omega^2 - V_{\omega,l}^2(r))R = 0 \tag{3}$$

where,

$$\frac{dr^*}{dr} = (1 - \frac{2M}{r})^{-1} \tag{4}$$

$$W_l^2(r) = -\omega^2 - \left(1 - \frac{2M}{r}\right)\left(m^2 + \frac{l(l+1)}{r^2} + \frac{2M}{r^2}\right) \tag{5}$$

Let us decompose the wave function of a stationary state into partial waves (4). We obtain a standard expression for the scattering amplitude:

$$f(\theta) = \frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)\exp(2i\delta_l)\,P_l(cos\theta); \quad \theta \neq 0 \quad \text{and angle:} \quad \theta(l) = -\frac{2d\delta_l}{dl} \tag{6}$$

Where $k = \sqrt{\omega^2 - \mu^2} = \omega v$ is the relativistic wave vector, while δ_l is the scattering phase , responding to asymptotic behavior of the radial function

$$R_{\omega,l}(r) = 2\sin\left(kr - \frac{\pi l}{2} + \delta_l\right) \quad \text{when } r \to \infty \tag{7}$$

and could became complex. Such complex behavior could take place in systems with absorption. Obviously in systems with events horizons, as in cases of Black Holes, the absorption could be small, when the scattered particles meat a height potential barrier, and could be great when such a barrier is small, or is absent at all. In the last case, meanwhile, could take place an over barrier reflection, which lead to reversing of the magnitudes of absorption/reflection ratio. If one suppose that an absorption take place on the events horizon $r = \frac{2GM}{c^2}$ the scattering phase would became complex, while the imaginary part would describe the absorption of the partial wave inside a horizon. These imaginary part of the phase shift is needed for the calculation of the absorption cross section partial and total. If the imaginary part of the phase shift is changing sign, a particles generation can take place.

$$W_l(r) = (\omega^2)^2 - \left(1 - \frac{2M}{r}\right)\left(m^2 + \frac{L^2}{r^2}\right) = \omega^2 - V_{\omega,l}^2 \tag{8}$$

where the following substitution has been used:

$$l(l+1) \to \left(l+\frac{1}{2}\right)^2 \equiv L^2 \text{ for } l \gg 1 \qquad (9)$$

The function $V^2_{\omega,l}$ could be considered an effective potential energy of a particle in the field of a black hole, in a quasiclassical limit. In a strongly classical limit , when $L < L_c$, where

$W_{L_c}(r_{max}) = 0$, a fall of particles into the back hole take place , while the state with $L = L_c$ is responding to capture of particles on unstable circular orbit [2, 16]. If $Im\delta_l > 0$ such a way that $|\exp(2i\delta_l)| \ll 1$. For particles with $L > L_c \gg 1$ the tunneling is weak, and in this case the JWKB method is allowed to calculate the scattering phases , see [5, 16]. The real part of scattering phases can be calculated from the well known Quantum mechanical formula:

$$Re\delta_l = \int_{r_2}^{\infty} \sqrt{W_l(r)}dr^* - \int_{r_2}^{\infty} \sqrt{k^2 - \frac{L^2}{r^2}}dr \qquad (10)$$

The second integral in the formula above gives us the phases of free motion; r_2 means the biggest of 2 turning points: $W_L(r_{1,2}) = 0$, while $r_2 > r_1$. The first integral in the last formula for the real part of the scattering phase contains a characteristic for long range potentials diverging part of the phase:

$$\sim \frac{2k^2M+\kappa}{k} ln2kr , \quad \text{where } \kappa = m^2M \qquad (11)$$

Scattering for intermediary angles, backward and forward glory scattering

Let us mention that it is reducing to elliptic integrals in the case of Black Holes metrics such as Schwarzschild and analytic calculations are possible only in the limiting cases: $L \gg L_c$ and $L \geq L_c$. This is opposite to the well studied cases of Coulombian (Newtonian)-like fields non relativistic or relativistic. First case is corresponding to very great impact parameters when the scattering angle has a Rutherford-like shape with corrections, while the second case corresponds to spiral orbits near the unstable circular orbit. This case was studied carefully by C. DeWitt–Morette and coauthors [4] for massless fields and arbitrary spins. A generalization of this case for mass spinless particles has been given in [5]. A general formula for backward glory scattering for mass particles of any spin would have a form:

$$\frac{d\sigma}{d\Omega} = \frac{\pi}{4} \frac{R_g^2}{1-\left({mc^2}/{E}\right)^2} \frac{ER_g^2}{\hbar c} AJ_{2s}^2 \left(sin\theta \frac{ER_g^2}{2\hbar c} f\right) \qquad (12)$$

where s-is particles spin, while A and f are dimensionless functions of the velocity of particles at infinity, which have the form $f^2 = \frac{(1+8v^2)^{3/2}+(8v^4+20v^2-1)}{2v^2}$. This function has the limits 16 for $v = 0$ and 27 for $v = 1$.The function A is a little bit more complicated:

$$A(v) = \frac{4f^3(6\beta)^3\beta^{1/2}exp\left(-2\pi\beta^{1/2}\right)}{(2-\beta)\left[\left(3\beta^{1/2}+(2\beta-1)^{1/2}\right)\right]^4}\text{, where } \beta = \left(1-\frac{12(mM)^2}{L_c^2}\right)^{1/2} = \left[\frac{5+44v^2+32v^4-3\left(1+8v^2\right)^{3/2}}{8(1-v^2)}\right] \quad (13)$$

The limits of the function $A(v)$ are 17 for $v = 0$ and 4.06 at $= 1$, while the limits of the function β are 0.5 for $v = 0$, and 1 for $v = 1$. As a particular result of interest: the backward glory scattering for photons is exactly 0, as was shown by B. Mashhoon [9] due to interference of the waves with opposite polarizations. As we told before the forward glory scattering which also decreases fast with rotation number around a black hole is masked by Rutherford- type (but gravitational on nature) long range scattering. But if we could avoid Newtonian-like gravitational field here also glories could be observed for particles and waves.

The case of spiral motion could present a special interest, as it comprises orbits, with many spirals around a Black Hole. The same orbits could generate instabilities of kinematic nature, bistabilities, attractors and stochastic chaos. A small variation of the impact parameter could dramatically change the fate of a particle: it could reflect and escape to infinity, or it could fall irreversibly into the Black Hole if the particle is spinless (Bose-particle) one, or be captured in a second potential well, between the horizon of the Black Hole and the unstable orbit radius. The problem is how the limiting cases rely with elliptic integrals? If this relation is one caused by physical parameters of the problem, then the calculation of the elliptic integrals should give the same result as the application of a perturbation scheme, which could be named iterative , or Born. The well known from the Quantum mechanics in flat spacetime Born method is not applicable rigorously speaking because of very strong gravitational Black Hole's field. It is important to understand when, the Born approximation could be applied. This is rather a mathematical artificial method of solving elliptic integrals.

I shall develop below a perturbative method of solving the main General-relativistic- Quantum mechanical formula (14), which was used before by Sanchez for massless particles. This formula is important since it includes also the tortoise coordinate and implicitly take into account the curvature of the space-time. As I told before, the main advantage of this methods is: they allow to avoid computations of long elliptic integrals. The scattering amplitude could be represented as a sum:

$$f(\theta) = f_{L \gg L_c}(\theta) + f_{L_c}(\theta) + f_{L<L_c}(\theta) \quad (14)$$

The first part of the amplitude represents the contribution of partial waves with great impact parameters and it causes the Rutherfordian tail of the scattering amplitude. Corrections to this amplitude due to relativistic gravitational effects would be taken into account below.

The magnitude of the spiral cross sections decreases significantly with increasing number of spiral, so that one spiral cross section obviously dominates.

The third part represent partially diffraction phenomena and spiral cross section, which could be important only at angles $\theta \to \pi$. Diffraction phenomena on Black Holes could be treated as diffraction on Black (absorbing) Nuclei, see: [10] and [13] . The scattering amplitude will be:

$$f_{abs}(\theta) = \frac{iL_c}{k\theta} J_1(L_c\theta), \;\; L_c \equiv l_c + \frac{1}{2}; \quad L_c\theta \ll 1 \qquad (15)$$

It is easy to seen , that the absorption scattering amplitude at small angles will be masked by Rutherford-like scattering amplitude, while at greater angles it would have an oscillating character with a slowly decreasing amplitude. In spite of the formula (17) is applicable rigorously speaking for angles $\theta \ll \frac{1}{L_c}$ the authors of [10, 13]. has found diffraction maxima for electrons scattered by black nuclei for angles $2 < \theta < 3$. Similar diffraction phenomena can be found by numerical calculations on Black Holes. For $\frac{GmM}{hc} > 1$ the maxima are deformed from their obvious diffraction shape, while in the opposite case, say $\frac{GmM}{hc} \approx 0.2$, the maxima are very like to diffraction ones.

Calculation of phase integrals and scattering amplitudes for small angles scattering

In this first case , when $L > L_c$, the term $\sim r^{-3}$ in the effective Regge-Wheeler function (respectively into the effective potential) could be examined as a perturbation. The physical reason for such an approximation consist in the smallness of the short range interaction $\sim r^{-3}$ as compared with centripetal term and especially with Coulombian-like (Newtonian-like) term which goes as $\sim r^{-1}$.

This allows us to calculate the exact phase integral (14) without elliptic functions and integrals. Let us represent the Regge-Wheeler function in a form:

$$W_l(r) = W_L^0(r) + W_L^1(r) \qquad (16)$$

$$W_L^{(0)}(r) = k^2 + \frac{2\kappa}{r} - \frac{L^2}{r^2}, \text{ with } L = l + 1/2 \qquad (17)$$

and $$W_L^1(r) = \frac{2ML^2}{r^3} \qquad (18)$$

In this case the integral (13) takes the form

$$Re\delta_l = \left(2kM + {}^{\kappa}/_{k}\right) ln2kr + \delta_l^0 + \delta_l^1 \qquad (19)$$

where

$$\delta_l^0 + \left(2kM + \frac{\kappa}{k}\right) ln2kr = \int_{r_2}^{\infty} \frac{\sqrt{W_L^0(r)}}{(1 - {}^{2M}/_{r})} dr - \int_{r_c}^{\infty} \sqrt{k^2 - \frac{L^2}{r^2}}\, dr \qquad (20)$$

$$\delta_l^1 \cong \frac{1}{2} \int_{r_2}^{\infty} \frac{W_L^1(r)dr}{(1 - \frac{2M}{r})\sqrt{W_L^0}} \qquad (21)$$

A perturbative method of calculation of the phase integral (20-21) was developed in [11,12]. But it is sensible to calculation of the root r_2 of the Regge-Wheeler function $\omega^2 - V^2_{\omega,l}(r)$. Let us consider another method of calculation of the phase integral. It is perturbative also, but not sensible to the roots. I shall consider here the term

$\frac{-2m^2M}{r}$ as a perturbation, while the term $\frac{2ML^2}{r^3}$ would be considered exactly. The phase integral except the free phases would have the form

$$\delta_l^{II} = \int_{r_2}^{r^*} \frac{\sqrt{\omega^2 - m^2 - \left(1-\frac{2M}{r}\right)\frac{L^2}{r^2} + \frac{2\mu^2 M}{r}}}{1-\frac{2M}{r}} \approx v \int_{r_2}^{r^*} \frac{\sqrt{\omega^2 - \left(1-\frac{2M}{r}\right)\frac{L^2}{v^2 r^2}}\,dr}{1-\frac{2M}{r}} + \frac{1}{v}\int_{r_2}^{r^*} \frac{m^2 M dr}{(r-2M)\sqrt{\omega^2 - \left(1-\frac{2M}{r}\right)\frac{L^2}{v^2 r^2}}} \quad (22)$$

The following rule:

$$\delta_l^{II} + \int_{r_2}^{r^*} \sqrt{k^2 - \frac{L^2}{r^2}}\,dr = \int_{r_2}^{r^*} \sqrt{W_l}\,dr^* - \int_{r_2}^{r^*} \sqrt{k^2 - \frac{L^2}{r^2}}\,dr \quad (23)$$

holds.

The integral (22) is including free phases, as well as logarithmically divergent for $L \gg 1$ terms.

As obviously $v = \sqrt{1 - \frac{m^2}{\omega^2}}$. The first integral in the right part of the equation (24) was calculated by Sanchez in the article [13], but the parameter $\frac{L^2}{v^2} \equiv \mathcal{L}^2$ of Sanchez. Let us preserve below only terms of the order~ $\frac{1}{L}$. The result of the calculations of the phase shift is:

$$\delta_l^{II} \cong -2kMLnL - kM + \frac{15\pi}{8}\frac{k^2M^2}{L} \quad (24)$$

The total scattering phase is

$$\delta_l = -\left(2kM + \frac{\kappa}{k}\right) lnL - kM + \frac{3\pi}{8}\frac{(5\omega^2 - m^2)}{L} \quad (25)$$

The elastic scattering amplitude, which is very important for interference effects account is:

$$f(\theta) = \frac{2\left(2kM + \frac{\kappa}{k}\right)}{\omega v \theta^2} \sqrt{1 + \frac{3\pi}{4}\frac{(5\omega^2 - m^2)M^2\theta}{\left(2kM + \frac{\kappa}{k}\right)^2}} \exp\left\{i\left[-2\left(2kM + \frac{\kappa}{k}\right) ln\left(\frac{2\left(2kM + \frac{\kappa}{k}\right)}{\theta}\right) + 2\left(2kM + \frac{\kappa}{k}\right) - 2\omega v M + \frac{3\pi}{8}\frac{(5\omega^2 - m^2)M^2}{\left(2kM + \frac{\kappa}{k}\right)}\theta - \frac{\pi}{4}\right]\right\} \quad (26)$$

The classical scattering angle will be

$$\theta(b) = -\frac{2d\delta_l}{dl} = \left(1 + \frac{1}{v^2}\right)\frac{R_G}{b} + \frac{\pi}{4}\frac{5\omega^2 - m^2}{\omega^2 - m^2}\left(\frac{R_G}{b}\right)^2 \quad (27)$$

which is Einsteinain one [1] with a correction, coinciding with one calculated by methods of classical mechanics on Black Hole background [18].

General relativistic corrections and contributions of nonlinear and nonlocal scattering effects

Such effects are due to own Energy-momentum tensor of the scattered (by a Schwarzshild centre) field. There is need to mention that the case of an electrically charged particle is very distinguished from a case of an uncharged particle. External gravitational field can be disturbed by the mass of the scattered particle as well as (evidently) by the electric charge of the scattered particle. Estimations for an electrically charged particle in an external gravitational field has been given by De Witt and Brehme (1960), De Witt and DeWitt-Morette (1964), while generalizations for mass particle and Newtonian external field has been given in [17].

For an electrically charged particle the DeWitt-Brehme-deWitt-Westpfahl , corrected by my (and Collins-Delbourgo-Williams [18]) general relativistic contribution cross section will be:

$$\frac{d\sigma}{d\Omega} = \left[\frac{R_G(2k^2+m^2)}{4k^2 sin^2\left(\frac{\theta}{2}\right)}\right]^2 + \frac{3\pi}{16}\frac{(5k^2+4m^2)R_G^2}{k^2\theta^3} - l_{Pl}^2\frac{\alpha^2 M}{m}\frac{\pi}{32}\frac{3k^2+2m^2}{k^2 sin^3\left(\frac{\theta}{2}\right)} \quad (28)$$

Similar effects appears in double Pulsar gravitational radiation, which were examined by Damour, but they are due to effect of gravitational waves on the pulsar orbit (1983). For mass and uncharged particles the estimations by Westpfahl show, that nonlinear effects appear in the same order of the scattering amplitude as corrections to Einstein effect $\Delta\theta \sim {R_G^2}/{b^2}$, i.e. the corrections to differential scattering cross section are proportional to $\frac{1}{\theta^3}$. The additional scattering phase would be of the order $\frac{k^2M^2}{L}$ but it is not calculated in [17]. There is no also there a calculation of the scattering amplitude. We can only suppose, that the increasing of the differential scattering cross section in a case of gravitational non-linear and non-local interaction means that it is equivalent to an additional attraction and would increase the differential scattering cross section and scattering angle.

The correction obtained in [17] is:

$$\left(\frac{d\sigma}{d\Omega}\right)_{nonlin} = \frac{{R_G}^2}{32 sin^3\left(\frac{\theta}{2}\right)}\frac{\pi(4+v^2)}{v^2} \cong \frac{R_G^2}{4\theta^3}\frac{\pi(4+v^2)}{v^2} = \frac{R_G^2}{4\theta^3}\frac{\pi(5\omega^2-m^2)}{\omega^2-m^2} \quad (29)$$

The additional scattering angle is:

$$sin\left(\frac{\theta}{2}\right) = \frac{3\pi}{8}\left(\frac{M}{b}\right)^2\frac{4+v^2}{v^2} = \frac{3\pi}{32}\left(\frac{R_G}{b}\right)^2\frac{5\omega^2-m^2}{\omega^2-m^2}, \quad \text{or} \quad (\Delta\theta)_{nonlin} \cong \frac{3\pi}{16}\left(\frac{R_G}{b}\right)^2\frac{5\omega^2-m^2}{\omega^2-m^2} \quad (30)$$

which is exactly the same as [27], calculated above. Then the total scattering cross section for a scalar electrically charged particle by a Schwarzschild Black Hole for small angles will be

$$\frac{d\sigma}{d\Omega} = \left[\frac{R_G(2k^2+m^2)}{4k^2 sin^2\left(\frac{\theta}{2}\right)}\right]^2 + \frac{3\pi}{8}\frac{(5k^2+4m^2)R_G^2}{k^2\theta^3} - l_{Pl}^2\frac{\alpha^2 M}{m}\frac{\pi}{32}\frac{3k^2+2m^2}{k^2 sin^3\left(\frac{\theta}{2}\right)} \quad (31)$$

If $Mm \leq 2 \cdot 10^{-5} {M_{Pl}}^2$ and $0.5 \cdot 10^{-5} \frac{mM}{M_{Pl}^2} \leq sin\left(\frac{\theta}{2}\right) \leq 1$ the correction due to electromagnetic self action could be comparable with the Einsteinian angle for angles not equal to 0. At the same time there are no chances to observe general relativistic correction at small angles. Only glories or diffraction effects could distingue them from Einsteinian (twice amplified Rutherfordian shape scattering) . Otherwise the total cross section would depends on the phase shift between the general relativistic phase correction (32) and nonlinear and nonlocal self-action phase, which is contributing by an additional

$$\Delta\theta_{nlin}(b) = \frac{3\pi}{16}\frac{5\omega^2-m^2}{\omega^2-m^2}\left(\frac{R_G}{b}\right)^2$$

So that these effects would change the correction. The total scattering angle is

$$\theta(b) = \left(1+\frac{1}{v^2}\right)\frac{R_G}{b} + \frac{\pi}{2}\frac{5\omega^2-m^2}{\omega^2-m^2}\left(\frac{R_G}{b}\right)^2 - \left(1+\frac{1}{v^2}\right)\frac{R_G}{b}\frac{\pi\alpha^2}{16}\frac{M_{Pl}^2}{mM} \qquad (32)$$

The first term is Einsteinian angle for mass particles. The second is a next general relativistic and gravitational self action correction ([18], [11-12], [17]) and the last is electromagnetic self-action correction [19, 17] . Let us mention that electromagnetic self action acts as repulsion, or adding a charge to the test mass the scattering angle would decrease from its Einsteinian value, while general relativistic correction and nonlinear gravitational self-action had an additional attractive character and increase the scattering angle.

Conclussions

The calculation of scattering phases, amplitudes, differential and total cross sections by Black Holes background , when they are described by exact metrics (and no weak field approach, or calculation of diagrams involving gravitons and other elementary quanta) for mass particles can be made by a combination of a JWKB method and perturbation technics, which allow to avoid hard calculation of elliptic integrals. No any approximation of the potential is allowed, as some approximations could give errors up to 19%. Results obtained agree in the limits with known results for massless particles.

References

1. Einstein, A., Erklarung der perihelion bewegung, Ann. Phys., Sitzungsber. preuss.Akad. Wiss., vol. 47, No.2, pp. 831-839, 1915; Die grundlage des allgemeinen Relativitatstheorie, - Ann. Phys., 49, N7, 769-822
2. D. Radu, I. Merches and D. Tatomir, Free and interacting quantum fields. First order Processes on electromagnetic and Gravitational backgrounds. World Scientific, 2018,714 pp.
3. Sir Charles Darwin, 1959, Proc Roy. Soc., 1959, **249**,180;
4. Misner Ch. W., Thorne K.S., Wheeler J.A., Gravitation, 1973

5. K.W. Ford, J.A. Wheeler (1959), Ann. Phys.,1959, vol.7,p239; 259; J.A. Wheeler, Quaderno, Acad. Naz. Dei Lincei (1971), N. 157, p. 133 (Appendix).
6. Tian-Rong Zhang & Cécile DeWitt-Morette, Phys. Rev. Lett., 1984, **52**, p.2313; deWitt-Morrete, C., Nelson, B., 1984, **D29**, 1663; Anninos, P., DeWitte-Morette, Cecile, Matzner, R.A., Yioutas P., Zhang T.R. , Phys. Rev.1992, **D46** , 4477; Cartier, P., deWitt-Morette, C., Functional Integration, 2010, Cambridge, Cambridge University Press.
7. Gaina, A.B., Kochorbe, F.G., JETP, vol.65, p.211.
8. Gaina, A.B., Sov. Phys. JETP, vol. 69(1), p.13
9. Mashhoon, B., Phys. Rev. D., 1974, **10,** p.1059
10. Pomeranchuk, I.Ya., Nekotoryie voprosy teorii yadra (Some problems of the nuclear theory, M., Gostekhizdat, 1945
11. Gaina, A.B., VINITI (Moscow) 1970-80, 1980; PhD Thesis, Moscow University, 1981 (see: http://articles.adsabs.harvard.edu/cgi-bin/get_file?pdfs/MoVIN/1970/1980MoVIN1970...20G.pdf)
12. Gaina, A.B., PhD work, Quantum particles in Kerr-Newman fields, Moscow, 1981. (see: http://inspirehep.net/record/1490271/files/Gaina%20PhD%20thesis.pdf) ; Izv vuz. Fizika, vol. 31(8), p.33 [Engl transl: Soviet Physics Journal, 1988, vol.31, p.544]
13. Sanchez, N., J. Math Phys., 1976 ,**17,** 688; Phys. Rev., 1977, **D16**, p.937; 1978, **D18**, 1030
14. Unruh, W., Phys. Rev, 1976, **D14**, 3251;
15. Gaina, A.B., Chizhov, G.A.,Vestn. Mosk Un-ta, vol.24,No.2, p.3, [Engl. Transl : Mosc. Univ. Phys. Bull., 1983, **38,** No.2, p.3]
16. Landau , L.D., Lifshits, E.M., Fields Theory; Landau, L.D., Lifshits, E.M., Quantum mechanics, London, Pergamon
17. Westpfahl, K., Fortschr. Phys., 1985, vol. **33**(8), p.417
18. Collins, P., Delbourgo, D., Williams, R. , J. Phys., 1973, A. **6**, p.161
19. DeWitt, B., Brehme, D, Ann Phys, 1960, 9, 220; deWitt B, deWitt-Morette, C., Physics (N.Y.) , 1964, **1**, p. 3
20. Sibgatullin, N.R., JETP, 1974, **39**, p.579
21. Daudé, Th., Nicoleau, Fr., Mem. Am. Math. Soc. 2017, 1170, III-VI, 113 p