Spin-3/2 fields in D-dimensional Reissner-Nordström black hole spacetimes

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Abstract

We study spin-3/2 fields in higher dimensional spherically symmetric black holes. The analysis of these fields in curved spacetimes is usually carried out using the Newman-Penrose formalism. Here we develop a more direct approach using eigenspinor-vectors on spheres, to separate out the angular parts of the fields in the Rarita-Schwinger equation. Moreover, for non Ricci-flat spacetimes, the covariant derivative has to be modified to the supercovariant derivative to maintain gauge invariance. In doing so we obtain the radial equations of the corresponding gauge-invariant variable, from which we evaluate the quasinormal spectra as well as the absorption probabilities using the WKB approximation and the asymptotic iteration method.

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In supergravity theories [1, 2] the gravitino is described by a spin-3/2 field. The equations of motion of these spin-3/2 fields are given by the Rarita-Schwinger equation:

$$\gamma^{\mu\nu\alpha}\nabla_\nu\psi_\alpha = 0, \tag{0.1}$$

where

$$\gamma^{\mu\nu\alpha} \equiv \gamma^{[\mu}\gamma^{\nu}\gamma^{\alpha]} = \gamma^{\mu}\gamma^{\nu}\gamma^{\alpha} - \gamma^{\mu}g^{\nu\alpha} + \gamma^{\nu}g^{\mu\alpha} - \gamma^{\alpha}g^{\mu\nu} \tag{0.2}$$

is antisymmetric product of Dirac gamma matrices, ∇_ν is the covariant derivative, and ψ_α is the spin-3/2 field. In four dimensional black hole spacetimes the Rarita-Schwinger equations are usually analyzed in the Newman-Penrose formalism. However, this formalism cannot be extended to higher dimensions in a straightforward way. In our previous works [3, 4] we have tried an alternative approach to deal with spherically symmetric black hole cases. Using a complete set of eigenspinor-vectors on N-spheres, we were able to separate the radial and angular parts of the Rarita-Schwinger equation. In this work we would like to extend our considerations to charged black hole spacetimes.

The Rarita-Schwinger equation is invariant under the gauge transformation

$$\psi'_\alpha = \psi_\alpha + \nabla_\alpha\varphi, \tag{0.3}$$

where φ is a gauge spinor, provided that the background spacetime is Ricci-flat [3, 4]. This is not the case for charged black holes, nor for black holes in de Sitter or anti-de Sitter spaces. To maintain the gauge symmetry in those cases it is necessary to modify the covariant derivative into the so-called the "supercovariant derivative". This is done by adding terms related to the cosmological constant and the electromagnetic field of the black hole. Here we shall concentrate on charged Reissner-Nordström black holes in asymptotically flat spacetimes, where in the following section we shall show in detail how the supercovariant derivative is constructed in this case.

Using the supercovariant derivative we are able to obtain the Rarita-Schwinger equation for spin-3/2 fields in Reissner-Nordström black hole spacetimes. Since the spacetime is still spherically symmetric, it is possible, as in our previous works, to derive the radial equations for each component of the spin-3/2 field using eigenspinor-vectors on the N-sphere. However, the component fields are not gauge invariant, while the physical fields should be. Hence we shall, as in Ref. [4], construct a combination of the component fields, which is gauge invariant. That is, we shall use the same gauge invariant variables and work out the corresponding radial equations.

As the aim in this work is to study spin-3/2 fields near a Reissner-Nordström black hole, we will focus on how the charge Q of the black hole affects the behavior of the fields. This is done by studying the quasi-normal models (QNMs) associated to our fields, where QNMs are characterized by their complex frequencies. The real parts of the frequencies represent the frequencies of oscillations, while the imaginary parts the decay constants of damping. These QNMs are uniquely determined by the parameters of the black hole [5], where in order to determine these QNMs we will use the WKB and improved Asymptotic Iterative Method (AIM), where these methods and how to implement them are given in Refs. [4, 6–8]. Finally, using the WKB method we are able to obtain the absorption probabilities associated to our spin-3/2 fields, which can give us an insight into the grey-body factors and cross-sections of the black hole.

Our main results on the QNM frequencies and the absorption probabilities of the spin-3/2 fields are for both non-TT eigenmodes and TT eigenmodes. First we looked at the non-TT modes, where we found that when the charge of the black hole Q is increased from 0 to M the maximum value of the effective potential increases, while the peak of the potential becomes sharper. The result of this on the quasi-normal frequencies is that both the real part and the magnitude of the imaginary part will increase. For the absorption probability the curve will shift to higher energy when Q is increased. However, this trend will be reversed from the $j = 3/2$ and $D = 7$ case upwards, such that the maximum value of the potential will instead decrease as Q is increased. For higher dimensions, more and more modes would have this behavior.

For the TT-modes, the situation seems to be simpler. Firstly we need to mention that the effective potential in this case is not the same as the one for Dirac fields in the same spacetime, this is due to the extra terms present in the supercovariant derivative. A typical example of the change of the potential with changing Q. When Q is increased, the maximum value of the potential decreases and the peak broadens. Hence, the corresponding real part of the quasi-normal frequency will decrease, and so too the magnitude of the imaginary part. For the absorption probability the curve will then shift to lower energies as Q is increased. This is opposite to the trend observed for the non-TT cases when the dimension is $D < 7$.

We have found that for higher dimensions, and especially for the charge Q near the extremal value, the effective potential will develop another maximum. We believe that this is also a property of the potentials in high enough dimensions. The shape of the potential will become more complicated due to the appearance of more maxima and minima. This will pose difficulties to the WKB approximations and the AIM we used to evaluate the QNMs, as well as the absorption probabilities. This problem is more prominent for larger values of Q, especially for the extremal cases.

Since our method is applicable to spherically symmetric spacetimes, the immediate applications would be to consider spin-3/2 fields for Schwarzschild and Reissner-Nordström black holes in de Sitter and anti-de Sitter spaces. Charged black holes in anti-de Sitter spaces are particularly interesting because of their relevance to the ground state of supergravity. With these studies we may have a general discussion of fermionic QNMs in spherically symmetric spacetimes, such as those that were done for bosonic fields, see Refs. [9, 10]. We are also interested in working out the absorption cross-sections in our subsequent works. To do that we need to find the degeneracies of the eigenspinor-vectors on the N-sphere. One should be able to do that by following the method of Camporesi and Higuchi developed for Dirac spinors [11].

[1] A. Das and D. Z. Freedman, Nucl. Phys. B **114**, 271 (1976).

[2] M. T. Grisaru, H. N. Pendelton, and P. van Nieuwenhuizen, Phys. Rev. D **15**, 996 (1977).

[3] C. H. Chen, H. T. Cho, A. S. Cornell, G. Harmsen, and W. Naylor, Chin. J. Phys. **53**, 110101 (2015).

[4] C. H. Chen, H. T. Cho, A. S. Cornell, and G. Harmsen, Phys. Rev. D **94**, 044052 (2016).

[5] R. A. Konoplya and A. Zhidenko, Rev. Mod. Phys. **83**, 793 (2011).

[6] R. A. Konoplya, Phys. Rev. D **68**, 024018 (2003).

[7] H. T. Cho, A. S. Cornell, Jason Doukas, T.-R. Huang, and Wade Naylor, Adv. Math. Phys. **2012**, 281705 (2012).

[8] S. Iyer and C. M. Will, Phys. Rev. D **35**, 3621 (1987).

[9] V. Cardoso, J. P. S. Lemos and S. Yoshida, J. High Energy Phys. **0312** 041 (2003).

[10] V. Cardoso, J. P. S. Lemos and S. Yoshida, Phys.Rev. D **69**, 044004 (2004).

[11] R. Camporesi and A. Higuchi, J. Geom. Phys. **20**, 1 (1996).