No-Go theorems for ekpyrosis from ten-dimensional supergravity

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In this work, we discuss whether the new ekpyrotic scenario can be embedded into ten-dimensional supergravity. We use that the scalar potential obtained from flux compactifications of type II supergravity with sources has a universal scaling with respect to the dilaton and the volume mode. Similar to the investigation of inflationary models, we find very strong constraints ruling out ekpyrosis from analysing the fast-roll conditions. We conclude that flux compactifications tend to provide potentials that are neither too flat and positive (inflation) nor too steep and negative (ekpyrosis).

Keywords: Ekpyrosis; Type II string theory; Flux compactifications; Moduli.

1. Introduction

The strong no-go theorems which exclude tree-level de Sitter compactifications under a few simple assumptions with or without negative tension objects such as orientifold planes have been much explored because of the possible cosmological and phenomenological interests. However, the no-go theorems for ekpyrotic scenario which is alternative to inflation model in string theory is much less extensive. Our motivation for the present work is to improve this situation.

It is the purpose of this work to give a No-Go theorem of the ekpyrotic scenario in a ten-dimensional supergravity model which is low energy limit of a string theory. Section 2 describes the potential of scalar field for ekpyrotic scenario and the way it derives the four-dimensional effective theory. We discuss the approach to the effective action in more detail. The No-Go theorem of the ekpyrosis thus given by the string theory is discussed. We also investigate the detailed properties of these models, their embedding in a string theory and their viability. For simplicity, we do not consider D-branes and the associated moduli except for the volume modulus (breathing mode) of internal space although the analysis would not be different. Finally, section 3 provides a brief summary and an outlook to future developments.

2. No-Go theorem of the ekpyrotic scenario in the type II theory

In this section, we consider compactifications of the type II theory to four-dimensional spacetime on compact manifold Y. The ten-dimensional low-energy effective action for the type II theory takes the form [1]

$$S = \frac{1}{2\bar{\kappa}^2}\int d^{10}x\sqrt{-g}\left[\mathrm{e}^{-2\phi}\left(R + 4g^{MN}\partial_M\phi\partial_N\phi - \frac{1}{2}\left|H\right|^2\right) - \frac{1}{2}\sum_p \left|F_p\right|^2\right] - \sum_p\left(T_{\mathrm{D}p} + T_{\mathrm{O}p}\right)\int d^{p+1}x\sqrt{-g_{p+1}}\,\mathrm{e}^{-\phi}\,, \quad (1)$$

where $\bar{\kappa}^2$ is the ten-dimensional gravitational constant, g , R denote the determinant, the Ricci scalar with respect to the ten-dimensional metric g_{MN} , respectively, ϕ is the scalar field, H is the NS-NS 3-form field strength, F_p are the R-R p-form field strengths ($p = 0, 2, 4, 6, 8$ for type IIA, and $p = 1, 3, 5, 7, 9$ for type IIB) that are sourced by D-branes, and $T_{\mathrm{D}p}$ ($T_{\mathrm{O}p}$) is the Dp-brane (Op-plane) charge and tension. Although there are Chern-Simons terms in the ten-dimensional action, these are essentially independent of the dilaton and the scale of the background metric. Hence, we will not consider them.

To compactify the theory to four dimensions, we consider the a metric ansatz of the form[1]

$$ds^2 = q_{\mu\nu}dx^\mu dx^\nu + g_{ij}dy^i dy^j = q_{\mu\nu}dx^\mu dx^\nu + \rho\, u_{ij}(\mathrm{Y})dy^i dy^j \,, \tag{2}$$

where ρ is breathing mode (volume modulus of the compact space), x^μ denote the coordinates of four-dimensional spacetime, y^i are local coordinates on the internal space Y, and $q_{\mu\nu}$, $u_{ij}(\mathrm{Y})$ are the metrices of four-dimensional spacetime, six-dimensional internal space, respectively. We assume that $q_{\mu\nu}$, $u_{ij}(\mathrm{Y})$ depend only on the coordinates x^μ, y^i , respectively. Since we factored out the overall volume modulus ρ of the internal space in the ten-dimensional metric (2), the modulus ρ is related to the total physical volume of the internal space v_6 and the volume of Y space $v(\mathrm{Y})$ as

$$\rho = \left[\frac{v_6}{v(\mathrm{Y})}\right]^{1/3}, \qquad v_6 = \int d^6y\sqrt{g_6}\,, \qquad v(\mathrm{Y}) = \int d^6y\sqrt{u}\,, \tag{3}$$

where g_6 and u denote the determinant of the metric g_{ij} and $u_{ij}(\mathrm{Y})$, respectively. The volume modulus ρ is chosen such that the metric $u_{ij}(\mathrm{Y})$ of the internal space is normalized $v(\mathrm{Y}) = 1$.

Now let us consider the four-dimensional effective action S_{E} in the Einstein frame after we integrate over the internal space Y. It is especially interesting to understand the dynamics of moduli at negative potential energy. If there are nontrivial fluxes in the background, one notes that these make uplifting the moduli potential to positive energy. In this work, we consider the moduli potential without field strengths and D-branes. Then we obtain the four-dimensional effective action in the Einstein frame:

$$S_{\mathrm{E}} = \int d^4x\sqrt{-\bar{q}}\left[\frac{1}{2\kappa^2}\bar{R} - \frac{1}{2}\bar{q}^{\mu\nu}\partial_\mu\bar{\tau}\,\partial_\nu\bar{\tau} - \frac{1}{2}\bar{q}^{\mu\nu}\partial_\mu\bar{\rho}\,\partial_\nu\bar{\rho} - V(\bar{\tau}\,,\bar{\rho})\right], \tag{4}$$

where κ^2 is the four-dimensional gravitational constant, $V(\bar{\tau}\,,\bar{\rho})$ denotes the moduli potential, and we have defined the dilaton modulus[1]

$$\tau = \mathrm{e}^{-\phi}\rho^{3/2}\,, \qquad \bar{\rho} = \sqrt{\frac{3}{2}}\kappa^{-1}\ln\rho\,, \qquad \bar{\tau} = \sqrt{2}\kappa^{-1}\ln\tau\,. \tag{5}$$

We have performed a conformal transformation on the four-dimensional metric $q_{\mu\nu} = (\bar{\kappa}/\tau\kappa)^2\bar{q}_{\mu\nu}$, where $\bar{q}_{\mu\nu}$ is the four-dimensional metric in the Einstein frame. $\bar{q}$ and $\bar{R}$ in the four-dimensional action (4) are the Ricci scalar and the determinant constructed from the metric $\bar{q}_{\mu\nu}$, respectively. Orientifold planes occupy $(p-3)$-dimensional internal space due to extending our four-dimensional universe. Then, the contribution of Op-plane ($p \geq 3$) to moduli potential survives. The moduli potential of four-dimensional effective theory is given by

$$V(\bar{\tau}\,,\bar{\rho}) = V_{\rm Y} + \sum_p V_{{\rm O}p} = -A_{\rm Y}\exp\left[-\kappa\left(\sqrt{2}\bar{\tau} + \frac{\sqrt{6}}{3}\bar{\rho}\right)\right]R({\rm Y}) \\ - \sum_p A_{{\rm O}p}\exp\left[-\kappa\left\{\frac{3\sqrt{2}}{2}\bar{\tau} + \frac{\sqrt{6}}{6}(6-p)\bar{\rho}\right\}\right]\int d^{p-3}x\sqrt{g_{p-3}}\,, \quad (6)$$

where $R({\rm Y})$ denotes the Ricci scalar constructed from the metric $u_{ij}({\rm Y})$ and $A_{\rm Y}$, $A_{{\rm O}p}$ are coefficients to scale with numbers of Op-planes. These coefficients in general depend on the function of the moduli of the internal space Y .

When the potential form for the ekpyrotic scenario gives the negative and steep, the fast-roll parameters for the ekpyrosis have to obey [2]

$$\varepsilon_{\rm f} \equiv \kappa^2\frac{V^2}{\left(\partial_{\bar{\tau}}V\right)^2 + \left(\partial_{\bar{\rho}}V\right)^2} \ll 1\,, \qquad |\eta_{\rm f}| \equiv \left|1 - \frac{V\left(\partial_{\bar{\tau}}^2V + \partial_{\bar{\rho}}^2V\right)}{\left(\partial_{\bar{\tau}}V\right)^2 + \left(\partial_{\bar{\rho}}V\right)^2}\right| \ll 1\,. \quad (7)$$

This is analogy with the standard slow-roll parameters in inflation. The potential form satisfying the condition (7) gives the ekpyrotic period of slow contraction before the big bang.

Now we consider IIA compactifications on an internal space (2), namely positive curvature and Ricci flat spaces, involving orientifold planes, and discuss the No-Go theorem for ekpyrotic scenario. The analysis will focus on the behavior of the moduli potential in the volume modulus and dilaton. In order to present the no-go theorem using these fields, we have to still make sure that there are no steep directions of the scalar potential in the $(\bar{\tau}\,,\bar{\rho})$-plane. In such cases one can then study directions involving $\bar{\tau}$, $\bar{\rho}$, and finds that the scalar potential satisfies

$$\varepsilon_{\rm f} = \frac{V^2}{2}\left[\left(V_{\rm Y} + \frac{3}{2}\sum_p V_{{\rm O}p}\right)^2 + \frac{\left(V_{\rm Y} + V_{\rm O4} - V_{\rm O8}\right)^2}{3}\right]^{-1} > \frac{6}{31}\,, \quad (8)$$

where $p = 4, 6, 8$. The fast-roll parameter $\varepsilon_{\rm f}$ has the bound $\varepsilon_{\rm f} > 6/31$. This result does not depend on the choice of coefficients $A_{\rm Y}$ and $A_{{\rm O}p}$. The value of parameter $\varepsilon_{\rm f}$ is not much less than one, which is the contradiction with the fast-roll condition for ekpyrosis (7) . Hence, ekpyrosis is not allowed [3,4].

On the other hand, for type IIB compactifications in the ekpyrotic model, we have also seen that it is possible to obtain simple no-go theorems in the $(\bar{\tau}\,,\bar{\rho})$-plane if one includes orientifold planes and the curvature of the internal space. From the Eq. (7), we find a constraint of the fast-roll parameter $\varepsilon_{\rm f}$

$$\varepsilon_{\rm f} = \frac{V^2}{2}\left[\left(V_{\rm Y} + \frac{3}{2}\sum_p V_{{\rm O}p}\right)^2 + \frac{\left(2V_{\rm Y} + 3V_{\rm O3} + V_{\rm O5} - V_{\rm O7} - 3V_{\rm O9}\right)^2}{12}\right]^{-1} > \frac{1}{6}\,, \quad (9)$$

where $p = 3, 5, 7, 9$. Unfortunately, the form moduli potential is not steep again as $\varepsilon_{\rm f}$ parameter turns out to be large value[4]. Just as in the IIA analogue, one obtains the bound $\varepsilon_{\rm f} > 1/6$. If we choose different values for $A_{\rm Y}$ and $A_{{\rm O}p}$ in the moduli potential (6), we can find the same bound.

3. Discussions

In this work, we have studied the No-Go theorem of the ekpyrosis for string theory in a spacetime of ten dimensions. We gave a potential of the scalar fields in four-dimensional effective theory, in terms of the compactification with smooth manifold. The effective potential of two scalar fields can be constructed by postulating suitable emergent gravity, orientifold planes, and vanishing fluxes on the ten-dimensional background. The scalar potential depends only on two moduli: $\bar{\tau}$ and $\bar{\rho}$. In such a simple setting, one can show that $\varepsilon_{\rm f} > 6/31$ for IIA theory and $\varepsilon_{\rm f} > 1/6$ for IIB theory whenever $V(\bar{\tau}, \bar{\rho}) < 0$. It has been known for some time that the effective potential of scalar fields requires the fast-roll parameter to be small during the ekpyrotic phase. However, with the help of the tools developed in section 2, this is prohibited in a string theory with a compactification we have considered. Hence, the explicit nature of the dynamics has made it impossible to realize the ekpyrotic phase in the present study. This is consistent with results in Ref. 3. In order to embed ekpyrotic or cyclic models in a ten-dimensional supergravity we have investigated in this work, we may consider some ingredients, for instance, the dynamics of remaining moduli, higher curvature correction other than orientifold and flux. We have not attempted an explicit construction here, since that will take us beyond the scope of this study. A lot of study remains to be done in string theory before a cosmologically realistic case is treated.

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