The MESS of cosmological perturbations

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We introduce two new effective quantities for the study of comoving curvature perturbations ζ: the *space* dependent effective sound speed (SESS) and the *momentum* dependent effective sound speed (MESS) . We use the SESS and the MESS to derive a new set of equations which can be applied to multi-fields systems and modified gravity theories. We show that this approach is completely equivalent to the standard one and it has the advantage of requiring to solve *only one* differential equation for ζ instead of a system, without the need of explicitly computing the evolution of entropy perturbations. The equations are valid for perturbations respect to any arbitrary flat spatially homogeneous background, including any inflationary and bounce model.

As an application we derive the equation for ζ for multi-fields KGB models and show that observed features of the primordial curvature perturbation spectrum are compatible with the effects of an appropriate local variation of the MESS in momentum space. The MESS is the *natural* quantity to parametrize in a model independent way the effects produced on curvature perturbations by multi-fields systems and modified gravity theories and could be conveniently used in the analysis of LSS observations, such as the ones from the upcoming EUCLID mission or CMB radiation measurements.

Introduction The evolution of comoving curvature perturbations ζ has a fundamental importance in cosmology since it is the basis for the study of different phenomena such as the CMB ansotropy or LSS. When the Universe is dominated by more than a scalar field the standard approach consists in deriving an equation for ζ, with a time dependent sound speed, obtaining a new source term [1] compared to the single field case, related to the field perturbations, which is interpreted as entropy perturbation. In this paper we show that there exist a completely equivalent form of the equations which involves a space dependent effective sound speed (SESS) and no source term. The advantage of this new approach is that it allows to study the evolution of curvature perturbations using only one equation for ζ without the need to introduce the notion entropy perturbations or to integrate the system of differential equations for all the fields, as done in the standard approach [1]. The same SESS can be defined for any system which in the standard formalism has entropy perturbations, including modified gravity theories where intrinsic entropy can be present also in the single field case, as in Horndeski's [2, 3] theory for example. We show with some examples that the new equation in terms of the SESS reduces to the equation with entropy terms, and derive the general relation between entropy and SESS.

After defining the momentum dependent effective sound speed (MESS), an equation for ζ in momentum space is derived and it is shown that features of the spectrum of curvature perturbations can be explained as variations of the MESS in momentum space. We also derive the equation for ζ for some single and multi-fields modified gravity scalar theories belonging to the Horndeski's family.

Derivation of the equationn for comoving curvature perturbations In order to derive an equation for comoving curvature perturbations ζ we manipulate the perturbed Einstein's equations, assuming this general form for the scalar perturbations of the metric and of the energy momentum (EM) tensor in absence of anisotropies

$$ds^2 = -(1+2A)dt^2 + 2a\partial_i B dx^i dt + + a^2 \left\{\delta_{ij}(1+2C) + 2\partial_i\partial_j E\right\} dx^i dx^j \,, \tag{1}$$

$$T^0{}_0 = -(\rho+\delta\rho) \quad , \quad T^0{}_i = (\rho+P)\partial_i(v+B)\,,$$
$$T^i{}_j = (P+\delta P)\delta^i{}_j\,. \tag{2}$$

where v is the velocity potential.

Note that the above equations are general, since no gauge has been chosen, and they can describe also multi-fields systems or modified gravity theories in absence of anisotropies. The comoving slices gauge, which we will call for brevity comoving gauge, is defined by the condition $(T^0{}_i)_c = 0$, where from now on we will be denoting with a subscript c quantities evaluated on comoving slices. We will use this notation for the metric and the perturbed EM tensor in the comoving gauge

$$ds^2 = -(1+2\gamma)dt^2 + 2a\partial_i\mu\, dx^i dt + + a^2 \left\{\delta_{ij}(1+2\zeta) + 2\partial_i\partial_j \nu\right\} dx^i dx^j \,, \tag{3}$$

$$(T^0{}_0)_c = -(\rho+\beta) \quad , \quad (T^i{}_j)_c = (P+\alpha)\delta^i{}_j\,. \tag{4}$$

where we have defined the gauge invariant quantities $\alpha = \delta P_c, \beta = \delta\rho_c, \gamma = A_c, \mu = B_c, \zeta = C_c, \nu = E_c$.

In the case of a single scalar field with a canonical kinetic term or in K-inflation the *comoving gauge* coincides with the *uniform field gauge*, also known as *unitary gauge*, but for multi-field systems they are *different*. From the gauge transformation

$$v + B \;\to\; v + B - \delta t\,, \tag{5}$$

we can derive the infinitesimal time translation taking to the comoving gauge, in which, $v_c + \mu = 0$

$$\delta t_c = \; v + B\,. \tag{6}$$

We can then define the following gauge invariant quantities

$$\alpha \;=\; \delta P + \dot{P}\delta t_c \quad , \quad \beta = \delta\rho + \dot{\rho}\delta t_c\,, \tag{7}$$
$$\gamma \;=\; A + \dot{\delta t_c} \quad , \quad \mu = B - a^{-1}\delta t_c\,, \tag{8}$$
$$\sigma \;=\; a\dot{E} - B + a^{-1}\delta t_c = a\dot{\nu} - \mu\,, \tag{9}$$
$$\zeta \;=\; C - H\delta t_c\,. \tag{10}$$

In the comoving gauge the Einstein's equations take the form [4, 5]

$$\frac{1}{a^2}\overset{(3)}{\Delta}[-\zeta + aH\sigma] = \frac{\beta}{2}\,, \tag{11}$$
$$\gamma = \frac{\dot{\zeta}}{H}\,, \tag{12}$$
$$-\ddot{\zeta} - 3H\dot{\zeta} + H\dot{\gamma} + (2\dot{H} + 3H^2)\gamma = \frac{\alpha}{2}\,, \tag{13}$$
$$\dot{\sigma} + 2H\sigma - \frac{\gamma + \zeta}{a} = 0\,, \tag{14}$$

where $\overset{(3)}{\Delta} \equiv \delta^{kl}\partial_k\partial_l$, a dot denotes a partial derivative respect to cosmic time t and we use a system of units in which $c = \hbar = M_{Pl} = 1$.

After replacing eq.(12) into eq.(13) we get the useful relation

$$\dot{\zeta} = -\frac{1}{2H\epsilon}\alpha\,, \tag{15}$$

where $\epsilon = -\dot{H}/H^2$. As we will see later this is the key equation to derive conservations laws for ζ and the second order differential equation in a closed form. In absence of anisotropies the Bardeen potentials [6] Φ_B and Ψ_B coincide, and we have

$$\sigma = \frac{\Phi_B + \zeta}{aH} \quad , \quad \gamma = \Phi_B + \partial_t(a\sigma)\,. \tag{16}$$

Replacing eq.(16) into eq.(11) we get

$$\frac{1}{a^2}\overset{(3)}{\Delta}\Phi_B = \frac{1}{2}\beta\,. \tag{17}$$

We define the space dependent effective sound speed (SESS) of comoving curvature perturbations according to the following relation

$$v_s^2(t, x^i) \equiv \frac{\alpha(t, x^i)}{\beta(t, x^i)}\,, \tag{18}$$

where α and β are the perturbed pressure and energy density in the comoving gauge.

We can now combine eqs.(15), (17) and (18) to obtain

$$\dot{\zeta} = -\frac{v_s^2}{a^2H\epsilon}\overset{(3)}{\Delta}\Phi_B\,. \tag{19}$$

Replacing eq.(16) into eq.(12) we get the useful relation between the comoving curvature perturbation and the Bardeen potential [6, 7]

$$\zeta = -\Phi_B + \frac{H^2}{\dot{H}}\left(\Phi_B + H^{-1}\dot{\Phi}_B\right) = \frac{H^2}{a\dot{H}}\partial_t\left(\frac{a\Phi_B}{H}\right)\,,$$

and combining it with the time derivative of eq.(19) we finally get

$$\partial_t\left(\frac{a^3\epsilon}{v_s^2}\dot{\zeta}\right) - a\epsilon\overset{(3)}{\Delta}\zeta = 0\,, \tag{20}$$

which can also be written as

$$\ddot{\zeta} + \frac{\partial_t(Z^2)}{Z^2}\dot{\zeta} - \frac{v_s^2}{a^2}\overset{(3)}{\Delta}\zeta = 0 \quad , \quad Z^2 = \frac{\epsilon a^3}{v_s^2}\,. \tag{21}$$

Equations (19), (20), and (21) together with the momentum space form derived later in eq.(48-49) are the main theoretical results of this paper. Note that eq.(19) is similar to the one obtained in [8], but without the entropy terms. We will explain this difference in more details in the section about two scalar fields, and more in general in the section about the relation between SESS and entropy.

Quite remarkably the equations above are valid for an arbitrary number of scalar fields, as long as the total perturbed EM tensor has no off-diagonal contributions, i.e. it has no anisotropic part. For multi-fields systems with total Lagrangians given by the sum of single field Lagrangians this condition is easily met, as shown in the following sections, and the equations are in fact valid for a wide class of multi-fields systems and modified gravity theories.

The equation is also valid for single field models with intrinsic entropy [3], such as the KGB theories [9]. For more general Horndeski's theories an additional term accounting for anisotropies in the EM tensor would be required, and we leave this case for a future work [3], but the definition of the SESS we have introduced in eq.(18) would not change.

Relation with entropy In the standard approach [4] entropy perturbations Γ are defined by

$$\alpha(t, x^i) = c_s(t)^2\beta(t, x^i) + \Gamma(t, x^i)\,, \tag{22}$$

where c_s is interpreted as sound speed and is a function of time only. This definition is invariant under the transformation

$$c_s^2 \to \tilde{c}_s(t)^2 = c_s(t)^2 + \Delta c_s(t)^2\,, \tag{23}$$
$$\Gamma \to \tilde{\Gamma}(t, x^i) = \Gamma(t, x^i) - \Delta c_s(t)^2\beta(t, x^i)\,, \tag{24}$$

where $\Delta c_s(t)^2$ is an arbitrary function of time only. The invariance of eq.(22) under these transformations shows that this definition of entropy is not unique, but if $c_s(t)^2$ is computed independently, for example from the action for the perturbations [10], this ambiguity should be resolved.

Combining eq.(22), (18) and (15) we get general relation between SESS and entropy

$$v_s^2 = c_s^2\left(1+\frac{\Gamma}{2\epsilon H\dot{\zeta}}\right)^{-1} . \qquad (25)$$

Replacing eq.(25) into eq.(19) and eq.(20) we get

$$\dot{\zeta} = -\frac{c_s^2}{a^2 H\epsilon}\overset{(3)}{\Delta}\Phi_B - \frac{\Gamma}{2H\epsilon}, \qquad (26)$$

$$\ddot{\zeta} + \frac{\partial_t z^2}{z^2}\dot{\zeta} - \frac{c_s^2}{a^2}\overset{(3)}{\Delta}\zeta + \frac{1}{z^2}\partial_t\left(\frac{a^3\Gamma}{c_s^2 H}\right) = 0, \qquad (27)$$

where $z^2 = 2a^3\epsilon/c_s^2$. These equations are in agreement with the general equation including the effects of entropy, derived in [11].

Application to two scalar fields In order to show that eq.(19) and eq.(20) are valid also for multifields systems we will consider the example of two scalar fields minimally coupled to gravity with Lagrangian

$$L = \sum_n^2 X_n + 2V(\Phi_1, \Phi_2),$$

where $X_n = g^{\mu\nu}\partial_\mu\Phi_n\partial_\nu\Phi_n$. In this section we will use this notation $\Phi_n(x^\mu) = \phi_n(t) + \delta\phi_n(x^\mu), \Phi = \Phi_1, \Psi = \Phi_2$. The components of the perturbed EM of the two scalar fields system, without gauge fixing, are

$$\delta T^0{}_0 = -\dot{\phi}\dot{\delta\phi} - \dot{\psi}\dot{\delta\psi} + A(\dot{\phi}^2+\dot{\psi}^2) - V_\phi\delta\phi - V_\psi\delta\psi,$$

$$\delta T^i{}_j = \delta^i_j\left[\dot{\phi}\dot{\delta\phi} + \dot{\psi}\dot{\delta\psi} - A(\dot{\phi}^2+\dot{\psi}^2) - V_\phi\delta\phi - V_\psi\delta\psi\right]$$

$$\delta T^0{}_i = \partial_i\left(-\frac{\dot{\phi}\delta\phi + \dot{\psi}\delta\psi}{a}\right), \qquad (28)$$

where we are denoting the partial derivatives as $V_\phi = \partial_\phi V(\phi,\psi)$ and $V_\psi = \partial_\psi V(\phi,\psi)$.

The comoving gauge is defined by the condition $(\delta T^0{}_i)_c = 0$, which implies that

$$\dot{\phi}U_\phi + \dot{\psi}U_\psi = 0, \qquad (29)$$

where $U_n = (\delta\phi_n)_c = \delta\phi_n + \dot{\phi}_n\delta t_c$ are the field perturbations in the comoving gauge, as defined in eq.(31), which are by construction gauge invariant. It is important to note that, contrary to the single field case, the *comoving gauge* for multi-fields systems *does not coincide* with the *uniform field gauge*, also known as *unitary gauge*, since there is not enough gauge freedom to set both fields perturbation to zero, i.e. in general it is impossible to use gauge transformation to choose a coordinate system in which $\delta\phi = \delta\psi = 0$. This is the origin of the *space dependency* of the SESS for multi-fields systems.

Under an infinitesimal time translation of the form $t \to t - \delta t$ the fields perturbations transform according to gauge transformation

$$\widetilde{\delta\phi} = \delta\phi + \dot{\phi}\delta t \quad , \quad \widetilde{\delta\psi} = \delta\psi + \dot{\psi}\delta t . \qquad (30)$$

From these equations we can find the time translation δt_c necessary to go to the comoving gauge, by imposing the comoving gauge condition $(\delta T^0{}_i)_c = 0 \to \dot{\phi}\widetilde{\delta\phi} + \dot{\psi}\widetilde{\delta\psi} = 0$, obtaining

$$\delta t_c = -\frac{\dot{\phi}\delta\phi + \dot{\psi}\delta\psi}{\dot{\phi}^2 + \dot{\psi}^2} . \qquad (31)$$

We can now compute explicitly gauge invariant quantities defined in the comoving gauge, such as the comoving field perturbation

$$U_\phi = \delta\phi - \dot{\phi}\frac{\dot{\phi}\delta\phi + \dot{\psi}\delta\psi}{\dot{\phi}^2+\dot{\psi}^2}, \; U_\psi = \delta\psi - \dot{\psi}\frac{\dot{\phi}\delta\phi + \dot{\psi}\delta\psi}{\dot{\phi}^2+\dot{\psi}^2},$$

and the comoving pressure and energy perturbations

$$\alpha = \delta P_c = \dot{\phi}\dot{U}_\phi + \dot{\psi}\dot{U}_\psi - \gamma(\dot{\phi}^2+\dot{\psi}^2) + \\ + (\ddot{\phi} + 3H\dot{\phi})U_\phi + (\ddot{\psi} + 3H\dot{\psi})U_\psi, \qquad (32)$$

$$\beta = \delta\rho_c = \dot{\phi}\dot{U}_\phi + \dot{\psi}\dot{U}_\psi - \gamma(\dot{\phi}^2+\dot{\psi}^2) + \\ - (\ddot{\phi} + 3H\dot{\phi})U_\phi - (\ddot{\psi} + 3H\dot{\psi})U_\psi . \qquad (33)$$

After replacing eq.(12) and the expressions for U_n into these equations we finally get

$$\beta = -\frac{\dot{\zeta}(\dot{\phi}^2+\dot{\psi}^2)}{H} - \frac{\Theta(\dot{\phi}^2+\dot{\psi}^2)}{2}, \; \alpha = -\frac{\dot{\zeta}(\dot{\phi}^2+\dot{\psi}^2)}{H},$$

where the function Θ is defined according to

$$\Theta = \left(\frac{\delta\phi}{\dot{\phi}} - \frac{\delta\psi}{\dot{\psi}}\right)\frac{\partial}{\partial t}\left(\frac{\dot{\phi}^2 - \dot{\psi}^2}{\dot{\phi}^2+\dot{\psi}^2}\right) . \qquad (34)$$

It is important to note that the above expression is gauge invariant, i.e.

$$\left(\frac{\delta\phi}{\dot{\phi}} - \frac{\delta\psi}{\dot{\psi}}\right) = \left(\frac{Q_\phi}{\dot{\phi}} - \frac{Q_\psi}{\dot{\psi}}\right) = \left(\frac{U_\phi}{\dot{\phi}} - \frac{U_\psi}{\dot{\psi}}\right) \qquad (35)$$

where

$$Q_\phi \equiv \delta\phi + \frac{\dot{\phi}}{H}C \quad , \quad Q_\psi \equiv \delta\psi + \frac{\dot{\psi}}{H}C, \qquad (36)$$

are the field perturbations in the flat gauge.

Assuming a classical field trajectory parametrized as $\psi(\phi)$ we can write Θ in this form

$$\Theta = 4\dot{\phi}\frac{\partial\psi}{\partial\phi}\frac{\partial^2\psi}{\partial\phi^2}\left[\left(\frac{\partial\psi}{\partial\phi}\right)^2 + 1\right]^{-2}\left(\frac{U_\psi}{\dot{\psi}} - \frac{U_\phi}{\dot{\phi}}\right) \qquad (37)$$

From the above expression we can see that in order for Θ to be different from zero the trajectory has to have non vanising first and second derivatives, i.e. there must be some turn in the field space.

Combining eqs.(18) with the expressions derived above for α and β we obtain the EES for the two scalar fields

$$v_s^2 = \left(1 + \frac{H\Theta}{2\dot{\zeta}}\right)^{-1} . \tag{38}$$

and replacing it into eq.(19) and eq.(21) we get

$$\dot{\zeta} = \frac{H}{a^2\dot{H}} \overset{(3)}{\Delta} \Phi_B - \frac{1}{2}H\Theta , \tag{39}$$

$$\ddot{\zeta} + \frac{\partial_t(z^2)}{z^2}\dot{\zeta} - \frac{1}{a^2} \overset{(3)}{\Delta} \zeta + \frac{1}{z^2}\partial_t\left(\frac{z^2 H\Theta}{2}\right) = 0 , \tag{40}$$

in agreement with [1, 8, 12], confirming that eq.(19), obtained for a generic EM tensor, is indeed valid also for multi-field systems. When the relation between the SESS and $\dot{\zeta}$ in eq.(38) is used explicitly the equation for ζ has a source term which is commonly interpreted as entropy [1, 8], while in the form of eq.(21) there is no entropy but the SESS is not only time but also space dependent. This example shows that the two descriptions are completely equivalent, and that the notion of entropy is in fact not really necessary, as long as the SESS is defined appropriately as in eq.(18), as a *space-time* dependent quantity.

The evolution of comoving curvature perturbations for multiple scalar fields systems could be studied without introducing any notion of entropy perturbation, by specifying an appropriate SESS $v_s(x^\mu)$. The advantage of this approach is that SESS can be used as an effective quantity without explicitly specifying the multi-fields model or solving the system of differential equations for the field perturbations Q_ϕ and Q_ψ, as done in the standard approach [1].

As we have shown for the particular case of two fields, for an arbitrary number of scalar fields it is enough to solve just equation (21), and this can be a substantial computational advantage in presence of several fields.

The SESS for generic multi-fields systems and particle production The Lagrangian for N fields system with standard kinetic term is

$$L = \sum_n^N X_n + 2V(\Phi_n) \tag{41}$$

where $X_n = g^{\mu\nu}\partial_\mu\Phi_n\partial_\nu\Phi_n$ and $\Phi_n(x^\mu) = \phi_n(t) + \delta\phi_n(x^\mu)$. Depending on the type of potential, particle production could occur [13, 14] and according to our approach this would correspond to a specific space dependent form of the EES $v_s(x^\mu)$. The effects of particle production on the curvature perturbations could be modeled phenomenologically, without specifying the potential, by considering different forms of the EES $v_s(x^\mu)$. The results derived in the previous section can be generalized to

$$v_s^2 = \left(1 + \frac{H\Theta}{2\dot{\zeta}}\right)^{-1} . \tag{42}$$

where the function Θ is proportional to

$$\theta_{ij} = \left(\frac{\delta\phi_i}{\dot{\phi}_i} - \frac{\delta\phi_j}{\dot{\phi}_j}\right)\frac{\partial}{\partial t}\left(\frac{\dot{\phi}_i^{\,2} - \dot{\phi}_j^{\,2}}{\sum_i^n \dot{\phi}_i^{\,2}}\right) , \; \Theta = \chi_N \sum_{i>j}^N \theta_{ij}$$

and χ_N is an appropriate combinational factor depending on the number of fields N.

Comparing with eq.(25) we get that $\Gamma = \epsilon H\Theta$, from which we can define $\Gamma_{ij} = \epsilon H\Theta_{ij}$ as the as the entropy due to the interaction between fields pairs, in terms of which the total interacting entropy is given by $\Gamma = \chi_N \sum_{i>j}^n \Gamma_{ij}$.

Application to multi-fields Horndeski's theory For single field Horndeski theory the decomposition of the perturbed EM tensor shows the presence of intrinsic entropy Γ^{int} [3]

$$\alpha = c_s^2(t)\beta + \Gamma^{int} \tag{43}$$

and in the particular case of KGB models with Lagrangian [9]

$$L_{KG}(\Phi, X) = K(\Phi, X) + G(\Phi, X)\Box\Phi ,$$

there is no anisotropy, so that equation (25) is valid with this definition of SESS

$$v_{KG}^2 = c_s^2\left(1 + \frac{\Gamma^{int}}{2\epsilon H\dot{\zeta}}\right)^{-1} . \tag{44}$$

The same result could also be generalized to multi-field KGB models with Lagrangian $L_{NKG} = \sum_n^N L_{KG}(\Phi_n, X_n)$, in which case the total entropy would be given by the sum of the intrinsic and interaction entropy as

$$\Gamma_{NKG} = \sum_i^N \Gamma_i^{int} + \chi_N \sum_{i>j}^N \Gamma_{ij} , \tag{45}$$

and the corresponding SESS is

$$v_{NKG}^2 = c_s^2\left(1 + \frac{\Gamma_{NKG}}{2\epsilon H\dot{\zeta}}\right)^{-1} . \tag{46}$$

Momentum dependent effective sound speed We define the momentum dependent effective sound speed (MESS) $\tilde{v}_k(t)^2$ as

$$\tilde{v}_k^2(t) \equiv \frac{\alpha_k(t)}{\beta_k(t)} , \tag{47}$$

and following a procedure mathematically similar to the one used to derive eq.(21) we can obtain this equation in momentum space [3]

$$\ddot{\zeta}_k + \frac{\partial_t(\tilde{Z}_k^2)}{\tilde{Z}_k^2}\dot{\zeta}_k + \frac{\tilde{v}_k^2}{a^2}k^2\zeta_k = 0 \quad , \quad \tilde{Z}_k^2 = \epsilon a^3/\tilde{v}_k^2 \quad (48)$$

It is important to note that the MESS $\tilde{v}_k(t)$ defined in eq.(47) is not simply the Fourier transform of the SESS $v_s(x^\mu)$ defined in eq.(18), because the product of the Fourier transforms of two functions is the transform of the convolution of the two functions.

After introducing the variable $u_k \equiv \tilde{Z}_k\zeta_k$ eq.(48) takes the form

$$\ddot{u}_k + \left(\tilde{v}_k^2k^2 - \frac{\ddot{\tilde{Z}}_k}{\tilde{Z}_k}\right)u_k = 0\,, \quad (49)$$

which reduces to the well-known Sasaki-Mukhanov equation when $\tilde{v}_k = 1$. In the case in which $\tilde{v}_k$ is not a function of time, equations (48) and (49) can be written as

$$\zeta_k'' + \frac{\partial_\eta(z^2)}{z^2}\zeta_k' + \tilde{v}_k^2k^2\zeta_k = 0\,, \quad (50)$$

$$u_k'' + \left(\tilde{v}_k^2k^2 - \frac{z''}{z}\right)u_k = 0\,, \quad (51)$$

where the prime denotes derivatives with respect to conformal time τ, and $z^2 = 2a^2\epsilon$.

One interesting application of this equation consists in computing the effects of the MESS on the spectrum of primordial curvature perturbations, motivated by the observations pointing to local deviations from power law [15–18]. In fig.(1) we plot the relative difference $\Delta P_\zeta/P_\zeta$ between the power spectrum of a slow-roll model compatible with observations and with $\tilde{v}_k = 1$, and that of model with the same parameters and the MESS given by

$$\tilde{v}_k = 1 + A_c\exp\left[-\left(\frac{k-k_0}{\sigma}\right)^2\right]\,, \quad (52)$$

where the scale $k_0 \approx 1.5\times10^{-3}Mpc^{-1}$ corresponds to the location of one of the most statistically significant features [18]. We compute the spectrum choosing the Bunch-Davies vacuum [19] for modes deeply inside the horizon

$$\zeta_k = \frac{1}{2a\sqrt{\epsilon\tilde{v}_k k}}e^{-i\tilde{v}_k k\tau}\,, \quad (53)$$

which has been normalized using the Wronskian condition for u_k. Due to the non trivial MESS the modes which leave the horizon around $\tau_0 = -1/k_0$ do not freeze right after $\tau_k = -1/k$ but can have some residual super-horizon evolution, and consequently the curvature perturbation equation (51) has to be integrated

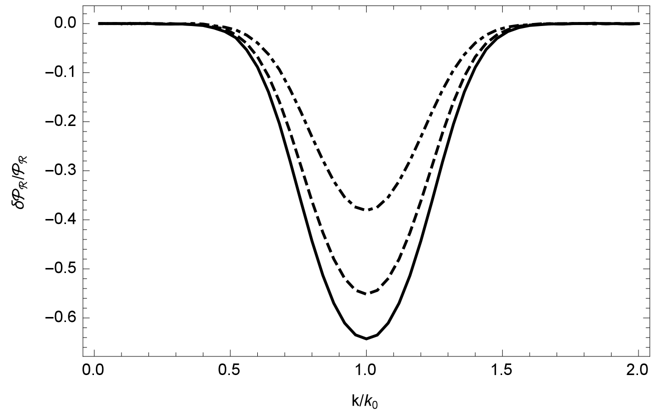

FIG. 1: The relative difference $\Delta\mathcal{P}_\zeta/\mathcal{P}_\zeta$ is plotted as a function of k/k_0. The solid, dashed and dot-dashed lines correspond $\sigma = 2.5\times10^{-1}k_0$ and $A_c = 4\times10^{-1}$, $A_c = 3\times10^{-1}$ and $A_c = 1.7\times10^{-1}$ respectively.

beyond the standard freezing time $-1/k$. As shown in fig.(1) the effects of the MESS on the curvature perturbation spectrum are qualitatively in agreement with the local features whose presence is supported by observations. A more systematic analysis could consist in fitting observational data using a general parametrization for the MESS, for example a piecewise cubic Hermite interpolating polynomial(PCHIP). Note that in general the MESS can be both momentum and time dependent, and from a phenomenological point of view another convenient parameter to use for a model independent analysis is $\tilde{Z}_k(\tau)$, which could be fitted with a double PCHIP, one for the time and the other for the momentum dependence, corresponding for example to an ansatz of the form $\tilde{Z}_k(\tau) = f(k)g(\tau)$.

The advantage of this approach is that it allows to study the effects of a wide class of theoretical models such as multi-fields systems, modified gravity theories or a combination of the two such as for the $nKGB$ model discussed previously, using just one equation. Once the MESS has been constrained by model a independent analysis of observational data the theoretical models can be compared to it.

Conclusions After introducing a new definition of the space dependent effective sound speed (SESS) and the momentum dependent effective sound speed (MESS) we have derived a set of new equations (19-21) and (51-49) for the comoving curvature curvature perturbations valid for any system with an EM tensor without anisotropic terms, including multi-fields systems, modified gravity theories, or a combination of the two. The approach we have developed is completely equivalent to the standard one, and it does not require any notion of entropy perturbation. The use of the SESS and the MESS simplifies significantly the calculation of the evolution of curvature perturbations, since it involves the solution of only one differential equation instead of a system.

The other important advantage is that it allows to make a model independent analysis of observational data using effective quantities without specifying the details of the underlying theoretical model. As an application we have shown that features of the primordial curvature perturbation spectrum can be modeled with an appropriate choice of the MESS, and we have derived the equation for comoving curvature perturbation for a multi-field modified gravity theory, the $nKGB$ model. In the future it will be interesting to derive a more general equation involving anisotropic terms in the EM tensor and to apply this new approach to the analysis of LSS observations and CMB data using a PCHIP parametrization for both the time and space dependency of the MESS.

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[1] C. Gordon, D. Wands, B. A. Bassett, and R. Maartens, Phys. Rev. D **63**, 023506 (2001), astro-ph/0009131.
[2] G. W. Horndeski, Int. J. Theor. Phys. **10**, 363 (1974).
[3] A. E. Romano and S. A. Vallejo, Work in preparation .
[4] H. Kodama and M. Sasaki, Prog. Theor. Phys. Suppl. **78**, 1 (1984).
[5] V. F. Mukhanov, H. A. Feldman, and R. H. Brandenberger, Phys. Rept. **215**, 203 (1992).
[6] J. M. Bardeen, Phys. Rev. **D22**, 1882 (1980).
[7] J. M. Bardeen, P. J. Steinhardt, and M. S. Turner, Phys. Rev. **D28**, 679 (1983).
[8] J. García-Bellido and D. Wands, Phys. Rev. D **52**, 6739 (1995), gr-qc/9506050.
[9] C. Deffayet, O. Pujolas, I. Sawicki, and A. Vikman, JCAP **1010**, 026 (2010), arXiv:1008.0048.
[10] J. M. Maldacena, JHEP **05**, 013 (2003), arXiv:astro-ph/0210603.
[11] S. Vallejo Peña, A. Enea Romano, A. Naruko, and M. Sasaki, ArXiv e-prints (2018), arXiv:1804.05005.
[12] F. Finelli and R. H. Brandenberger, Phys. Rev. **D62**, 083502 (2000), arXiv:hep-ph/0003172.
[13] A. E. Romano and M. Sasaki, Phys. Rev. **D78**, 103522 (2008), arXiv:0809.5142.
[14] D. J. H. Chung, E. W. Kolb, A. Riotto, and I. I. Tkachev, Phys. Rev. **D62**, 043508 (2000), arXiv:hep-ph/9910437.
[15] M. Benetti, S. Pandolfi, M. Lattanzi, M. Martinelli, and A. Melchiorri, Phys. Rev. **D87**, 023519 (2013), arXiv:1210.3562.
[16] K. Horiguchi, K. Ichiki, and J. Yokoyama, PTEP **2017**, 093E01 (2017), arXiv:1705.02770.
[17] P. Hunt and S. Sarkar, JCAP **1512**, 052 (2015), arXiv:1510.03338.
[18] Planck, P. A. R. Ade *et al.*, Astron. Astrophys. **594**, A20 (2016), arXiv:1502.02114.
[19] T. S. Bunch and P. C. W. Davies, Proc. Roy. Soc. Lond. **A360**, 117 (1978).