Cosmographic Analysis as framework to evaluate cosmological models

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By using a cosmographic analysis of the redshift data of type Ia supernovae, we are able to get the expansion of the scale factor, obtaining the current values of the Hubble, deceleration, jerk and snap parameters. Our data is then used to compare the fitness of various proposed alternative cosmological models. Since our method assumes only the validity of general relativity at the cosmic scale, along with the isotropy and homogeneity of the universe, they are very useful for comparison between different cosmological models, including the fitness of ΛCDM model. Our method is based on the order expansion of the scale factor present in the FRW metric and using a Monte Carlo integration to find the best fit order parameters of the scale factor to reproduce the observed data, we make use of parallel paradigm to improve the computational time behind the model. We find the known result an accelerated expansion of the universe. With access to better measurements of type Ia supernovae redshifts and more data, the cosmographic results will be significantly improved.

Keywords: Cosmography; Type Ia Supernovae; ΛCDM; deceleration parameter; Monte Carlo Integration

1. Introduction

Although there is almost complete agreement on the accelerated expansion of the universe[1], ΛCDM being the most favored cosmological model by the current data, there is no consensus to the current specific value of this decelerated expansion and to whether or not this model is still prevalent in the future[2]. The decelerated expansion q_0 is the parameter indicating the current rate of acceleration of the expansion of the universe, therefore, it is the first indicator of the fitness of any cosmological model, so a precise determination of its value is necessary for the diagnostic of any cosmological model. There recently have been plenty of research but there is no consensus for its current value[3].

We obtain the expansion terms of the scale factor in the FRW line element to test different cosmological models. We compare the expected value of the apparent magnitude in terms of the measured redshift to each supernova with the measured value of its apparent magnitude. This comparison is made by means of a likelihood ratio test to find the current values of the coefficients in the expansion of the factor scale that best fit the measured data.

Our interest is in comparing the results from our analysic with proposed cosmological models (as in ref.4 and ref.5). Even though cosmographic analysis has been previously studied (as in ref.6 and ref.7), we are performing the analysis using parallel computing what permits to make the expansion to higher orders than the usuals. We found that some results have significant changes when calculations are improved and how the cosmography works as a framework to assess cosmological models.

2. Friedman-Robertson-Walker and Cosmography

2.1. *Friedman-Robertson-Walker*

The line element for all the homogeneous and isotropic models of the universe is the Friedman-Robertson-Walker metric, shown in 1. Where $a(t)$ is the expansion factor of the universe, which gives us the rate at which the universe is expanding. This value depends on the content and matter-energy densities for the universe and it is theoretically found using the Friedman equation, which is obtained from the Einstein's field equations. The constant k is only determined by its sign, if $k < 0$ the universe is said to be open and the spatial hypersurfaces have negative constant curvature, if $k > 0$ the universe is said to be closed and spatial hypersurfaces have positive constant curvature, for $k = 0$ the universe is said to be flat with the spatial hypersurfaces being Euclidean with curved spacetimes.

$$ds^2 = c^2dt^2 - a(t)^2[\frac{dr^2}{(1-kr^2)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)] \tag{1}$$

2.2. *Cosmographic Analysis*

We proceed by using a cosmographic approach to determine the value of cosmological parameters. We seek to make a comparison of the measured values of the apparent magnitud (m) of the supernovae with the expected values given its measured redshift (z). The apparent magnitud is given by (2) in terms of the luminosity distance (d_L) and the absolute magnitude (M) which is known for supernovae to constant. Since the luminosity distance (3) is given in terms of the physical distance (r_0) between the source signal and the observer and the measured z we need to express r_0 in terms of the measured redshift. We do this by using the null geodesic in FRW and the cosmographic redshift. By the null geodesic we have:

$$m = 5\log\frac{d_L}{10} + M \tag{2}$$

$$d_L = (1+z)r_0a_0 \tag{3}$$

$$-c\int_{t_*}^{t_0}\frac{dt}{R(t)} = f(r_0) = \int_{r_0}^{0}\frac{dr}{\sqrt{(1-kr^2)}} \tag{4}$$

Where: $f(r_0) = \begin{cases} -\sin^{-1}(r_0) & (k=+1) \\ -r_0 & (k=0) \\ -\sinh^{-1}(r_0) & (k=-1) \end{cases}$

We expand the scale factor in FRW:

$$R(t) = R(t_0)[1 + H_0(t-t_0) - \frac{1}{2!}q_0 H_0^2 (t-t_0)^2 + \frac{1}{3!} j_0 H_0^3 (t-t_0)^3 + \frac{1}{4!} s_0 H_0^4 (t-t_0)^4 + ...]$$

and using the cosmological redshift relation with the scale factor expansion[8], we obtain the flight time from the source to us ($T \equiv t_0 - t_*$ where t_* is the time at which the signal was emitted) as a function of the measured redshift.

$$z + 1 = \frac{R(t_0)}{R(t_*)}$$

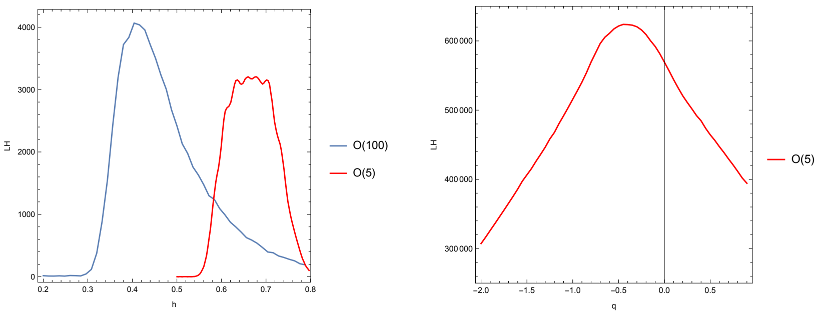
$$\frac{R(t_0)}{R(t_*)} = 1 + H_0 T + \frac{2+q_0}{2} H_0^2 T^2 + \frac{6(1+q_0)+j_0}{6} H_0^3 T^3 + \frac{24 - s_0 + 8j_0 + 36q_0 + 6q_0^2}{24} H_0^4 T^4 + ...$$

Numerically inverting:

$$T\left(\frac{z}{H_0}\right)^{-1} = 1 - \left[1 + \frac{q_0}{2}\right] z + \left[1 + q_0 + \frac{q_0^2}{2} - \frac{j_0}{6}\right] z^2$$

$$- \left[1 + \frac{3}{2} q_0 (1+q_0) + \frac{5}{8} q_0^3 - \frac{1}{2} j_0 - \frac{5}{12} q_0 j_0 - \frac{s_0}{24}\right] z^3 + ...$$

We solve the left side integral in eq. 4 with the expansion of the scale factor and substitute T in terms of z as found above. Therefore we are able to use the luminosity distance in terms of the redshift, to which we then employ a marginal likelihood ratio analysis in order to find the best fit values for the cosmological parameters. We are then able to use this data (from ref.6) to compare with any cosmological model.



(a) (b)

Fig. 1. Here are the results of our cosmographic analysis. (a)The Hubble parameter (b) The deceleration parameter.

In figure 1 (a), we are showing the results for the Hubble parameter for two orders of expansion. It is visible how when the order is higher the average is lower than the usual value, for 5 order we found $\langle H \rangle = 100 \langle h_0 \rangle = 67$ and for 100 order is $\langle H \rangle = 47$. The figure 1 (b) shows the deceleration parameter which in agreement with recent results is negative[3] $\langle q_0 \rangle = -0.48$.

3. Comparison of different models

We compare our obtained factor scale with two different theoretically proposed cosmological models. The first one[4] proposes the gravitational constant (G) and the cosmological constant (Λ) are not constants but instead functions of time. With the standard Friedman equations derivation and proposing they relate to each other by $G\rho = \frac{\eta\Lambda}{8\pi}$ with ρ the density of the perfect fluid and η a constant, there are two possible scale factors and deceleration parameters:

- Case A: $n \neq 0$

$$a(t) = (nlt + C_1)^{\frac{1}{n}} \tag{5}$$

$$q = n - 1 \tag{6}$$

- Case B: $n = 0$

$$a(t) = C_2 e^{lt} \tag{7}$$

$$q = -1 \tag{8}$$

The second one[5] proposes measuring the average expansion rate in a universe in which a set of spherically symmetric sub-regions expand in an accelerated way, *Average Expansion Rate Approximation (AvERA)*. It has the appeal that it conclusively resolves the tension between the measured values of the *Hubble constant* but the great drawback is that it is difficult to match with the homogeneity observed in the CMB.

We plot them in figure (2) with our obtained results to see how they compare to each other. We see that although one has several parameters to adjust to get greater similarity with our results can be more closely approximated by Case A of the given model, Case B is an exponential, so it starts at the value one and never approximates the rest of the curves.

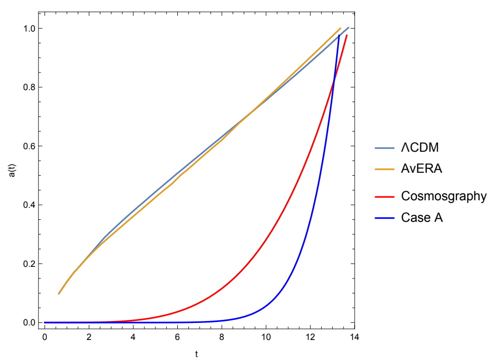


Fig. 2. Comparison of different models with the cosmographic result.

Acknowledgments

This work is partially supported by MICITT and CONICIT grant FI-0204-2012 and by grant 112-B8-107 of the Universidad de Costa Rica's Vicerrectoría de Investigación.

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