Dark energy from non-degenerate Higgs-vacuum

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ABSTRACT

Scalar fields are favorite among the possible candidates for the dark energy. Most frequently discussed are those with degenerate minima at $\pm\phi_{min}$. In this paper, a slightly modified two-Higgs doublet model is taken to contain the Higgs field(s) as the dark energy candidate(s). The model considered has two non-degenerate minima at $\phi_{\pm}$, instead of one degenerate minimum at $\pm\phi_{min}$. The component fields of one SU(2) doublet (ϕ_1) act as the standard model (SM) Higgs, while the component fields of the second doublet (ϕ_2) are taken to be the dark energy candidates (lying in the true vacuum). It is found that one *can* arrange for late time acceleration (dark energy) by using an SU(2) Higgs doublet, whose vacuum expectation value is zero, in the quintessential regime.

Introduction

There are three components of the total energy density of the Universe: (a) Non-relativistic matter; (b) Relativistic matter or radiation; (c) Dark energy. The part (c) is supposed to be causing the current observed accelerated expansion of the Universe. There are several candidates for the third component: (A) The cosmological constant, Λ[1–4]; (B) Modified gravity,[5,6]; (C) Scalar field models (e.g. quintessence, k-essence, tachyon field, phantom (ghost) field, dilatonic dark energy, Chaplygin gas)[4,7–9]; (D) Vector fields[10–19].

The dynamics of the Universe is described by the Einstein field equations (EFEs) which are

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}. \tag{1}$$

When explaining the accelerated expansion of the Universe from the Cosmological constant, Λ, one must also include the vacuum energy contribution from $T_{\mu\nu}$ since we know that because of the Lorentz invariance the energy momentum tensor in the vacuum takes the form

$$< T_{\mu\nu} >= -\rho_{vac}g_{\mu\nu}, \tag{2}$$

and hence the vacuum energy contribution cannot be neglected. On substituting eq. (2) in eq. (1) with simplification eq. (1) with no matter part can be written as

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda_{eff}g_{\mu\nu} = 0 \tag{3}$$

where $\Lambda_{eff} = \Lambda + \frac{8\pi G}{c^4}\rho_{vac}$. The ρ_{vac} contribution from by summing the zero point energies of all normal modes of some field with mass m to a wave number cutoff $\bar{\Lambda} \gg m$ gives a vacuum energy density[1]

$$\rho_{vac} = \int_0^{\bar{\Lambda}} \frac{4\pi k^2 \mathrm{d}k}{(2\pi)^3}\frac{1}{2}\sqrt{k^2+m^2} \approx \frac{\bar{\Lambda}^4}{16\pi^2}. \tag{4}$$

This gives a value of ρ_{vac} approximately $2\times 10^{71}\mathrm{GeV}^4$ where as the observed value of this ρ_{vac} is about $10^{-47}\mathrm{GeV}^4$ and mismatching is known as the Cosmological constant problem. The solution to this problem can be though in another way in which the spontaneous symmetry breaking is responsible for providing the such small value of the observed vacuum energy density. Here one assumes that the scalar field potential is of the form with its coefficient μ^2 being negative

$$V(\phi) = V_0 + \mu^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2$$

In this way, the vacuum energy, $\rho_{vac} = V_0 - \mu^4/4\lambda$, with the Cosmological constant, Λ, some how manage to give the observed effective Cosmological constant. But this method is very vivid and unnatural in constructing the observed vacuum energy. It has also been shown by Copeland et. al.[4] that the scalar field potential which can give rise to power-law expansion of the Universe are exponential potentials.

Scalar fields were first used by Alan Guth[20] to provide an inflationary solution to the horizon and flatness problems, and Andre Linde[21,22] to resolve the magnetic monopole and domain wall problems (arising from Guth's inflation) along with the earlier problems (for inflation see e.g.[23]). Similarly scalar, vector and tensorial fields can be used to explain the dark energy as being the dynamical vacuum energy[4,10].

The homogeneous and isotropic Universe is described by the Friedmann-Robertson-Walker (FRW) metric and its dynamics is described by the Friedmann equations which are

$$H^2 = \frac{8\pi G}{3}\rho - \frac{\kappa}{a^2} , \tag{5}$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6}\left(1+3\omega_{eff}\right)\rho , \tag{6}$$

where a is the scale factor, $H = \dot{a}/a$ is the Hubble parameter, ρ is the total energy density of the Universe and κ is the spatial curvature. In deriving the above equations, the barotropic equation of state $P = \omega\rho$ has been used; P being the pressure and ω the equation of state parameter. Here, and throughout, Planck units $\hbar = c = 1$ and $(8\pi G)^{-1/2} = M_P$ have been used and M_P has been taken to be 1. From eq. (6), we see that $\ddot{a} > 0$ (accelerated expansion of the Universe) when $\omega_{eff} < -\frac{1}{3}$.

Using the law of conservation of energy, it can easily be shown that

$$\rho \propto a^{-3(1+\omega)} . \tag{7}$$

Scalar fields are categorized as: (1) *Quintessence fields* ($-1 < \omega < -1/3$); (2) *Phantom fields* ($\omega < -1$). When $\omega = -1$ the energy density remains constant as the Universe expands.

The scalar fields generally used to explain accelerated expansion of the Universe are taken to be new fields (with no connection to Particle Physics whatsoever). However, the introduction of new fields with no experimental basis other than "explaining" one observed phenomenon is too reminiscent of the pre-relativistic aether. In our opinion, only if one can exclude fields contained in the standard model, or a minor extension of it, would it be justifiable to introduce such exotic proposals as phantom or quintessence fields, unless the new proposal simplifies the explanation *in the full detailed calculations*.

In this paper, we assume that the present state of the Universe is described by an inert uplifted double well two-Higgs doublet model in which both doublets lie in their true vacuum, and the vacuum expectation value (VeV) of the second doublet is zero. The results obtained favor the accelerated expansion of the Universe in the quintessence regime.

Method

Here we discuss the Uplifted double well two-Higgs doublet model (UDW-2HDM). We also discuss the conditions under which Higgs fields could be considered as dark energy fields.

Uplifted double well two-Higgs double model

The electroweak symmetry in the standard model (SM) of Particle Physics is broken spontaneously by the non-zero VeV of the Higgs field via Higgs mechanism. The Lagrangian which describes any model in Particle Physics is

$$\mathscr{L} = \mathscr{L}_{gf}^{SM} + \mathscr{L}_Y + \mathscr{L}_{Higgs}. \tag{8}$$

Here $\mathscr{L}_{gf}^{SM}$ is the $SU_C(3)\otimes SU_L(2)\otimes U_Y(1)$ SM interaction of the fermions and gauge bosons (force carriers), given by

$$\begin{aligned}\mathscr{L}_{gf}^{SM} &= -\tfrac{1}{4}G_{\mu\nu}G^{\mu\nu}-\tfrac{1}{4}W_{\mu\nu}W^{\mu\nu}-\tfrac{1}{4}B_{\mu\nu}B^{\mu\nu}+\bar{\psi}_L^i i\gamma^\mu\nabla_\mu^{EW}\psi_L^i+\bar{\psi}_R^i i\sigma^\mu\nabla_\mu^{EW}\psi_R^i \\ &\quad +\bar{\chi}_L^i i\gamma^\mu\nabla_\mu^{SM}\chi_L^i+\bar{U}_R^i i\sigma^\mu\nabla_\mu^{SM}U_R^i+\bar{D}_R^i i\sigma^\mu\nabla_\mu^{SM}D_R^i.\end{aligned} \tag{9}$$

The Yukawa interaction of fermions with the Higgs represented as $\mathscr{L}_Y$ is given as

$$\mathscr{L}_Y = -Y_{ij}^u\bar{\chi}_L^i\tilde{\phi}_1U_R^j-Y_{ij}^d\bar{\chi}_L^i\phi_1D_R^j-Y_{ij}^e\bar{\psi}_L^i\phi_1\psi_R^j-\text{h.c.} \tag{10}$$

here ψ_L^i are left handed leptons doublets, ψ_R^i are right handed leptons singlets, χ_L^i are left handed quark doublets, U_R^i and D_R^i are the right handed quark singlets. i runs from 1-3. ϕ is the SM Higgs doublet. The Higgs field Lagrangian, $\mathscr{L}_{Higgs}$, is

$$\mathscr{L}_{Higgs} = T_H - V_H, \tag{11}$$

here, T_H is the kinetic term of the Higgs field and V_H is the potential of the Higgs field. The kinetic and potential terms in UDW-2HDM are

$$T_H = (D_{1\mu}\phi_1)^\dagger(D_1{}^\mu\phi_1)+(D_{2\mu}\phi_2)^\dagger(D_2{}^\mu\phi_2)+\left[\chi(D_{1\mu}\phi_1)^\dagger(D_2{}^\mu\phi_2)+\chi^*(D_{2\mu}\phi_2)^\dagger(D_1{}^\mu\phi_1)\right], \tag{12}$$

and

$$\begin{aligned}V_H &= \rho_1\exp(\Lambda_1 m_{11}^2(\phi_1-\phi_{1_0})^\dagger(\phi_1-\phi_{1_0}))+\rho_3\exp\left(\frac{1}{2}\Lambda_3\lambda_1((\phi_1-\phi_{1_0})^\dagger(\phi_1-\phi_{1_0}))^2\right) \\ &\quad +\rho_2\exp(\Lambda_2 m_{22}^2(\phi_2-\phi_{2_0})^\dagger(\phi_2-\phi_{2_0}))+\rho_4\exp\left(\frac{1}{2}\Lambda_4\lambda_2((\phi_2-\phi_{2_0})^\dagger(\phi_2-\phi_{2_0}))^2\right) \\ &\quad +\lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2)+\lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)+\Big[m_{12}^2(\phi_1^\dagger\phi_2)+\frac{\lambda_5}{2}(\phi_1^\dagger\phi_2)^2+\lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) \\ &\quad +\lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2)+\text{h.c.}\Big],\end{aligned} \tag{13}$$

where

$$D_{1\mu}=\partial_\mu+i\frac{g_1}{2}\sigma_iW^i{}_\mu+i\frac{g_1'}{2}B_\mu,\qquad D_{2\mu}=\partial_\mu+i\frac{g_2}{2}\sigma_iW^i{}_\mu+i\frac{g_2'}{2}B_\mu,$$

$$\phi_i=\begin{bmatrix}\phi_i^+\\ \eta_i+i\chi_i+\nu_i\end{bmatrix},\qquad \phi_{i_0}=\begin{bmatrix}0\\ \tau_i\end{bmatrix}\qquad\text{and}\qquad \phi_i^\dagger=\begin{bmatrix}\phi_i^- & \eta_i-i\chi_i+\nu_i\end{bmatrix}.$$

The dimensions of the different quantities are $[\rho_i]^{-1}=[\Lambda_i]=[L]^4$, $[m_{ii}^2]=[L]^{-2}$, $[\phi_i]=[L]^{-1}$ and $[\lambda_i]=[L]^0$, where "L" denotes length. The Higgs fields ϕ_i^+, ϕ_i^-, η_i and χ_i are hermitian ($\phi_i^\pm$ are charged whereas the others are neutral), ν_i is the VeV of the doublet ϕ_i, ϕ_{i_0} in the potential is the true minimum of the field ϕ_i. Here we also assume that both the Higgs doublets are coupled with the gauge fields differently. In this way, we can suppress the interaction of dark energy Higgs with the gauge bosons. The shape of the potential in this model is shown in fig. (1).

Since we want the Higgs field (lying in the false vacuum) to live for not less than the current age of the Universe, a stable Higgs field is required. This can be achieved by imposing a discrete Z_2 symmetry.

There are two types of Z_2 symmetry breaking: 1) soft; and 2) hard. When Z_2 symmetry is broken by $(\phi_i^\dagger\phi_j)$ type terms then it is said to be softly broken and when Z_2 symmetry is broken by $(\phi_i^\dagger\phi_j)(\phi_k^\dagger\phi_l)$ type terms then Z_2 is said to be hardly broken. The term containing m_{12}^2 describes the soft Z_2 symmetry breaking, whereas the terms containing λ_6 and λ_7 describe the hard Z_2 symmetry breaking. In the absence of these terms along with no cross kinetic term i.e. $\chi=0$, the UDW-2HDM's Higgs Lagrangian has a perfect Z_2 symmetry[24]. There are two Z_2 symmetries corresponding to the UDW-2HDM doublets:

$$\text{I:}\qquad \begin{array}{lll}\phi_1 \longrightarrow -\phi_1, & \qquad & \phi_2 \longrightarrow \phi_2, \\ \phi_{1_0} \longrightarrow -\phi_{1_0}, & & \phi_{2_0} \longrightarrow \phi_{2_0};\end{array} \tag{14}$$

$$\text{II:}\qquad \begin{array}{lll}\phi_1 \longrightarrow \phi_1, & \qquad & \phi_2 \longrightarrow -\phi_2, \\ \phi_{1_0} \longrightarrow \phi_{1_0}, & & \phi_{2_0} \longrightarrow -\phi_{2_0}.\end{array} \tag{15}$$

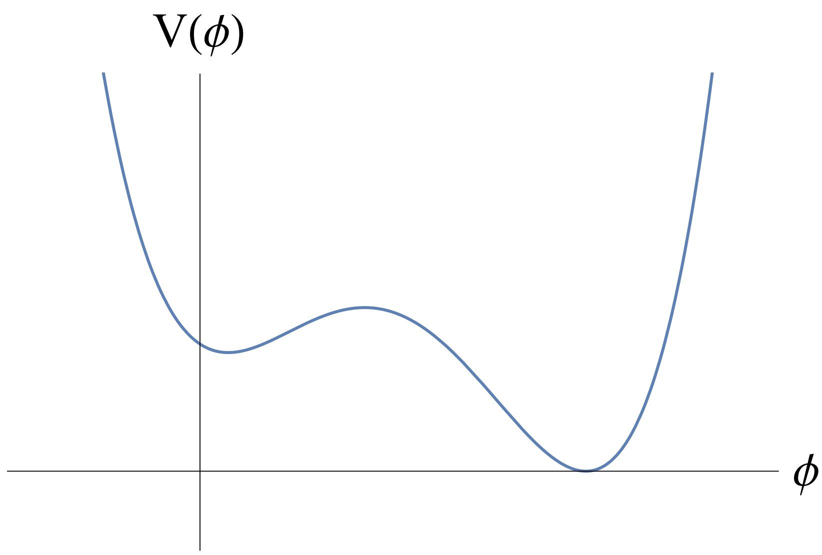

Figure 1. The uplifted double well potential.

Minimizing the Higgs potential

The extrema of the potential are found by taking

$$\frac{\partial V_H}{\partial \phi_1}\Big|_{\phi_1=\langle\phi_1\rangle} = \frac{\partial V_H}{\partial \phi_1^\dagger}\Big|_{\phi_1^\dagger=\left\langle\phi_1^\dagger\right\rangle} = 0 \qquad \text{and} \qquad \frac{\partial V_H}{\partial \phi_2}\Big|_{\phi_2=\langle\phi_2\rangle} = \frac{\partial V_H}{\partial \phi_2^\dagger}\Big|_{\phi_2^\dagger=\left\langle\phi_2^\dagger\right\rangle} = 0. \tag{16}$$

The most general solution of the conditions (16) is

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix} \qquad \text{and} \qquad \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} u \\ v_2 \end{pmatrix}.$$

The first solution of extrema has been taken to be similar to the Higgs vacuum in the SM and the second one is the most general that could occur. One needs to keep in mind that now $v^2 = v_1^2 + \left|v_2^2\right| + u^2$, where $v = 1/\sqrt[4]{2G_F{}^2} \approx 246$GeV is the VeV of the Higgs field in the SM.

If $u \neq 0$ the non-zero value of u will contribute to the “charged” type dark energy, which has not been observed. To avoid this, we choose $u = 0$. From the extrema conditions given by eq. (16), we can determine the values of v_1 and v_2[25–27], solving eq. (16) for the potential given by eq. (13) leads to

$$\begin{aligned}&\rho_1\Lambda_1 m_{11}^2(v_1-\tau_1)\exp\left(\Lambda_1 m_{11}^2(v_1-\tau_1)^2\right) + \rho_3\Lambda_3\lambda_1 v_1(v_1^2-\tau_1^2)\exp\left(\frac{\Lambda_3\lambda_1}{2}(v_1^2-\tau_1^2)^2\right)\\ &+m_{12}^2{}^* v_2 + (\lambda_3+\lambda_4+\lambda_5^*)v_1 v_2^2 + (\lambda_6+2\lambda_6^*)v_1^2 v_2 + \lambda_7^* v_2^3 = 0,\end{aligned} \tag{17}$$

$$\begin{aligned}&\rho_2\Lambda_2 m_{22}^2(v_2-\tau_2)\exp\left(\Lambda_2 m_{22}^2(v_2-\tau_2)^2\right) + \rho_4\Lambda_4\lambda_2 v_2(v_2^2-\tau_2^2)\exp\left(\frac{\Lambda_4\lambda_2}{2}(v_2^2-\tau_2^2)^2\right)\\ &+m_{12}^2 v_1 + (\lambda_3+\lambda_4+\lambda_5)v_1^2 v_2 + \lambda_6 v_1^3 + (2\lambda_7+\lambda_7^*)v_1 v_2^2 = 0.\end{aligned} \tag{18}$$

The solution of the above equations is practically impossible. Renormalizing, truncating the terms to fourth order in the potential given by eq. (13) and applying the minimization conditions given by eq. (16) we get,

$$\begin{aligned}&\rho_1\Lambda_1 m_{11}^2(v_1-\tau_1) + \rho_1\Lambda_1^2 m_{11}^4(v_1-\tau_1)^3 + \rho_3\Lambda_3\lambda_1 v_1(v_1^2-\tau_1^2) + m_{12}^2{}^* v_2\\ &+(\lambda_3+\lambda_4+\lambda_5^*)v_1 v_2^2 + (\lambda_6+2\lambda_6^*)v_1^2 v_2 + \lambda_7^* v_2^3 = 0,\end{aligned} \tag{19}$$

$$\rho_2\Lambda_2 m_{22}^2(v_2-\tau_2)+\rho_2\Lambda_2^2 m_{22}^4(v_2-\tau_2)^3+\rho_4\Lambda_4\lambda_2 v_2(v_2^2-\tau_2^2)+m_{12}^2 v_1$$
$$+(\lambda_3+\lambda_4+\lambda_5)v_1^2 v_2+\lambda_6 v_1^3+(2\lambda_7+\lambda_7^*)v_1 v_2^2=0. \tag{20}$$

As discussed before, Z_2 symmetry is very important in determining the phenomenology of the theory and with exact Z_2 symmetry the lightest Higgs field is stable, and hence becomes a candidate for dark energy. Imposing the Z_2 symmetry[24] we have,

$$\chi=0,\quad \mathrm{Re}(m_{12}^2)=\mathrm{Im}(m_{12}^2)=0,\quad \mathrm{Re}(\lambda_6)=\mathrm{Im}(\lambda_6)=0,\quad \mathrm{Re}(\lambda_7)=\mathrm{Im}(\lambda_7)=0. \tag{21}$$

Using eqs. (19, 20, 21) and imposing $\lambda_3+\lambda_4+\lambda_5=0$[7] (with $\lambda_3+\lambda_4+\lambda_5\neq 0$ the solution to the VeV contain complex part) we get four solutions for v_1 and v_2, which are:

$$v_1=\tau_1\ ,\qquad v_2=\tau_2\ ; \tag{22}$$

$$v_1=\tau_1\ ,\qquad v_2=\frac{2m_{22}^4\Lambda_2^2\rho_2\tau_2-\lambda_2\Lambda_4\rho_4\tau_2-\sqrt{\Xi_2}}{2\left(m_{22}^4\Lambda_2^2\rho_2+\lambda_2\Lambda_4\rho_4\right)}\ ; \tag{23}$$

$$v_1=\frac{2m_{11}^4\Lambda_1^2\rho_1\tau_1-\lambda_1\Lambda_3\rho_3\tau_1-\sqrt{\Xi_1}}{2\left(m_{11}^4\Lambda_1^2\rho_1+\lambda_1\Lambda_3\rho_3\right)}\ ,\qquad v_2=\tau_2\ ; \tag{24}$$

$$v_1=\frac{2m_{11}^4\Lambda_1^2\rho_1\tau_1-\lambda_1\Lambda_3\rho_3\tau_1-\sqrt{\Xi_1}}{2\left(m_{11}^4\Lambda_1^2\rho_1+\lambda_1\Lambda_3\rho_3\right)}\ ,\qquad v_2=\frac{2m_{22}^4\Lambda_2^2\rho_2\tau_2-\lambda_2\Lambda_4\rho_4\tau_2-\sqrt{\Xi_2}}{2\left(m_{22}^4\Lambda_2^2\rho_2+\lambda_2\Lambda_4\rho_4\right)}\ ; \tag{25}$$

where

$$\Xi_1=-4m_{11}^6\Lambda_1^3\rho_1^2-4m_{11}^2\lambda_1\Lambda_1\Lambda_3\rho_1\rho_3-8m_{11}^4\lambda_1\Lambda_1^2\Lambda_3\rho_1\rho_3\tau_1^2+\lambda_1^2\Lambda_3^2\rho_3^2\tau_1^2,$$

and

$$\Xi_2=-4m_{22}^6\Lambda_2^3\rho_2^2-4m_{22}^2\lambda_2\Lambda_2\Lambda_4\rho_2\rho_4-8m_{22}^4\lambda_2\Lambda_2^2\Lambda_4\rho_2\rho_4\tau_2^2+\lambda_2^2\Lambda_4^2\rho_4^2\tau_2^2.$$

The Higgs doublets, when the vacuum is given by eq. (22), are

$$\phi_1=\begin{bmatrix}\phi_1^+\\ \eta_1+i\chi_1+\tau_1\end{bmatrix},\qquad \phi_2=\begin{bmatrix}\phi_2^+\\ \eta_2+i\chi_2+\tau_2\end{bmatrix}. \tag{26}$$

If we choose $\tau_2=0$ then the fields $\phi_1^\pm$ and χ_1 become Goldstone bosons and the other fields become physical. With $\tau_2=0$, the Yukawa interactions are described by the interaction of ϕ_1 with fermions (as ϕ_2 does not couple to fermions but appears in loops). The Higgs and Yukawa Lagrangian in this case violate the Z_2 symmetry given by eq. (14) but respect that given by eq. (15) only when $\tau_2=0$. Thus, parity in ϕ_2 is conserved. This makes the lightest field of ϕ_2 stable. Without this stability, we would have a model for accelerated expanding Universe for a limited time depending upon the decay width (life time) of the dark energy field(s), because if field(s) can decay then it ceases to provide accelerated expansion after its decay to the other fields. It would then require fine tuning to make it stable enough till the current age of the Universe and then to "switch off". The masses of the fields in this vacuum are

$$\begin{aligned}
m_{\eta_1}^2 &= \rho_1\Lambda_1 m_{11}^2\ ,\\
m_{\eta_2}^2 &= \rho_2\Lambda_2 m_{22}^2+\frac{v^2}{2}(\lambda_3+\lambda_4+\lambda_5)\ ,\\
m_{\chi_2}^2 &= \rho_2\Lambda_2 m_{22}^2+\frac{v^2}{2}(\lambda_3+\lambda_4-\lambda_5)\ ,\\
m_{\phi_2^\pm}^2 &= 2\rho_2\Lambda_2 m_{22}^2+\lambda_3 v^2\ .
\end{aligned} \tag{27}$$

Results

Here we show that new CP odd and even Higgs field(s) can be give rise to an accelerated expanding Universe.

Higgs fields as dark energy

The Universe is homogeneous and isotropic at the cosmological scale, and its dynamics is described by the Friedmann equations given by eq. (5) and eq. (6). Equation (6) says that accelerated expansion will occur when $\omega_{eff} < -\frac{1}{3}$ where $\omega_{eff} = \Omega_{DE}\omega_{DE} + \Omega_R\omega_R + \Omega_M\omega_M$. For the field ϕ_2 to be the dark energy field, it must bring $\omega_{eff} < -\frac{1}{3}$ within the history of the Universe (in fact just now $Z \approx 0.32$ when $\Omega_{DE} = 0.7$ and $\Omega_{NR} = 0.3$). For this purpose we need to solve the Euler-Lagrange equations, which are

$$\partial_\mu \left(\frac{\partial(\sqrt{-g}\mathscr{L}_{Higgs})}{\partial(\partial_\mu \psi_i)} \right) - \frac{\partial(\sqrt{-g}\mathscr{L}_{Higgs})}{\partial \psi_i} = 0, \tag{28}$$

where ψ_i are different fields of doublets ϕ_1 and ϕ_2.

The Euler-Lagrange equations of motion in FRW Universe ($\sqrt{-g} = a(t)^3$) for the fields $\phi_2^{\pm}$, η_2 and χ_2 in this model are

$$\begin{aligned}&\ddot{\eta}_2 + 3\frac{\dot{a}}{a}\dot{\eta}_2 + \frac{1}{2}\eta_2 \Big(v^2(\lambda_3+\lambda_4+\lambda_5) + 2m_{22}^2\Lambda_2\rho_2 e^{m_{22}^2\Lambda_2(\chi_2{}^2+\eta_2{}^2+2\phi_2^{c2})/2} \\ &+\lambda_2\Lambda_4\rho_4 e^{\lambda_2\Lambda_4(\chi_2{}^2+\eta_2{}^2+2\phi_2^{c2})^2/8}\left(\chi_2{}^2+2\phi_2^{c2}\right)\Big) + \frac{1}{2}\lambda_2\Lambda_4\rho_4 e^{\lambda_2\Lambda_4(\chi_2{}^2+\eta_2{}^2+2\phi_2^{c2})^2/8}\eta_2{}^3 = 0,\end{aligned} \tag{29}$$

$$\begin{aligned}&\ddot{\chi}_2 + 3\frac{\dot{a}}{a}\dot{\chi}_2 + \frac{1}{2}\chi_2 \Big(v^2(\lambda_3+\lambda_4-\lambda_5) + 2m_{22}^2\Lambda_2\rho_2 e^{m_{22}^2\Lambda_2(\chi_2{}^2+\eta_2{}^2+2\phi_2^{c2})/2} \\ &+\lambda_2\Lambda_4\rho_4 e^{\lambda_2\Lambda_4(\chi_2{}^2+\eta_2{}^2+2\phi_2^{c2})^2/8}\left(\eta_2{}^2+2\phi_2^{c2}\right)\Big) + \frac{1}{2}\lambda_2\Lambda_4\rho_4 e^{\lambda_2\Lambda_4(\chi_2{}^2+\eta_2{}^2+2\phi_2^{c2})^2/8}\chi_2{}^3 = 0,\end{aligned} \tag{30}$$

$$\begin{aligned}\ddot{\phi}_2^c + 3\frac{\dot{a}}{a}\dot{\phi}_2^c \quad &+ \quad \phi_2^c \Big(v^2\lambda_3 + 2m_{22}^2\Lambda_2\rho_2 e^{m_{22}^2\Lambda_2(\chi_2{}^2+\eta_2{}^2+2\phi_2^{c2})/2} \\ &+ \quad \lambda_2\Lambda_4\rho_4 e^{\lambda_2\Lambda_4(\chi_2{}^2+\eta_2{}^2+2\phi_2^{c2})^2/8}\left(\chi_2{}^2+\eta_2{}^2+2\phi_2^{c2}\right)\Big) = 0,\end{aligned} \tag{31}$$

where c is $+$ or $-$. The energy density and pressure after expansion of UDW-2HDM Higgs Lagrangian for physical fields become

$$\begin{aligned}\rho_{\rm DE}/{\rm P}_{\rm DE} = \quad & \frac{1}{2}\dot{\chi}_2{}^2 + \frac{1}{2}\dot{\eta}_2{}^2 + \frac{1}{2}\dot{\phi}_2^{c2} \pm \Big(\rho_1 + \rho_3 + \frac{1}{2}v^2\lambda_3\phi_2^{c2} + \frac{1}{4}v^2(\lambda_3+\lambda_4+\lambda_5)\eta_2{}^2 \\ &+\frac{1}{4}v^2(\lambda_3+\lambda_4-\lambda_5)\chi_2{}^2 + \rho_2 e^{m_{22}^2\Lambda_2(\chi_2{}^2+\eta_2{}^2+2\phi_2^{c2})/2} \\ &+\rho_4 e^{\lambda_2\Lambda_4(\chi_2{}^4+2\chi_2{}^2\eta_2{}^2+\eta_2{}^4+4\chi_2{}^2\phi_2^{c2}+4\eta_2{}^2\phi_2^{c2}+4\phi_2^{c4})/8}\Big).\end{aligned} \tag{32}$$

For the cosmological evolution of the fields η_2, χ_2 and ϕ_2^c, the equations of motion (given by eqs. (29, 30, 31)) are solved with the Friedmann equations numerically in the flat Universe ($\kappa = 0$). The initial conditions used are $\eta_{2ini} = M_P$, $\chi_{2ini} = M_P$, $\phi^c_{2\,ini} = 0$, $\dot{\eta}_{2ini} = 0$, $\dot{\chi}_{2ini} = 0$ and $\dot{\phi}^c_{2\,ini} = 0$. The masses of the Higgs bosons in the analysis are taken to be $m_{\eta_2} = m_{\chi_2} = 1.0247 \times 10^{-59}$ GeV, to set the evolution of the energy densities as observed. Note that after imposing Z_2 symmetry there are five parameters which determine the masses of Higgs fields. With $m_{\eta_1} = m_{H_{SM}} = 125.7$GeV we have only one equations to determine the parameters values. Constraints coming from tree level MSSM has not been imposed. The Higgs fields masses in this analysis were calculated by eq. (27) by imposing $m_{\eta_1} = m_{H_{SM}} = 125.7$GeV and some the arbitrary choice of parameters since there was only one equations to determine all unknown free parameters. One important thing in our model is that the mass of the charged Higgs becomes arbitrary and thus any value of the charged Higgs fields will suffice.

The solution of the eqs. (29, 30, 31) along with Friedmann equations is shown below in the graphs.

During the initial stages $Z \gg 1$, the evolution of the Higgs fields, η_2, χ_2 and $\phi_2^{\pm}$, is frozen, as shown in fig. (2) and acts as a negligibly small vacuum energy component with $\omega = -1$. As time proceeds the Higgs fields begin to evolve towards the minimum of the potential, the energy density in the Higgs fields starts to dominate cosmologically (on the Hubble scale). During the evolution, ω_{Higgs} starts to increase and becomes > -1 as shown in fig. (3). In the very late (future) Universe ($Z \ll 0$), the fields come to rest at the minimum of the potential and a period with $\omega = -1$ is

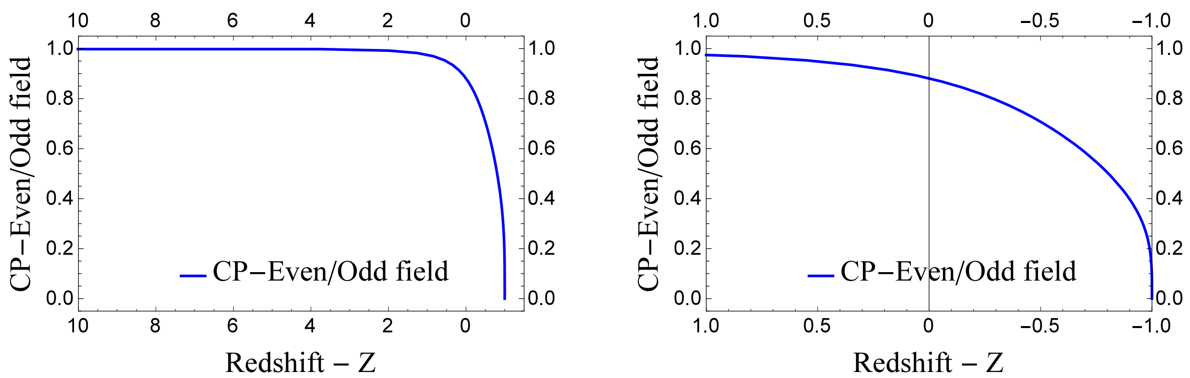

Figure 2. Higgs field as a function of redshift.

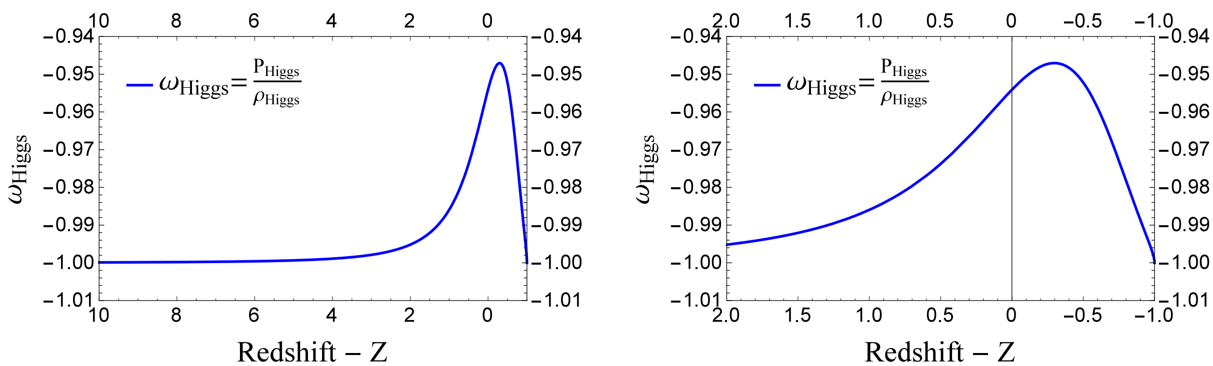

Figure 3. Effective equation of state parameter for Higgs fields ω_{Higgs}, as seen it starts with -1 then evolves towards quintessence regime after large enough time it comes back at -1.

reachieved to give an accelerating Universe similar to a pure cosmological constant. Since $\omega_{Higgs} \not> -1/3$ at any time in the evolution, after ω_{eff} becomes $< -1/3$, we get an exponentially accelerating Universe.

As discussed before, the Higgs field stability is provided by imposing Z_2 symmetry. The lightest Higgs fields, η_2 and χ_2, do not decay into any other Higgs field (since these fields are lighter than the SM-like and charged Higgs) or into fermions (since they do not couple to them at tree level).

The ω_{eff} in the fig. (4) starts from ≈ 0.167 (set by initial conditions $\Omega_{Higgs_{int}} = 0$ and $\Omega_{NR_{int}} = \Omega_{R_{int}} = 0.5$; NR: non-relativistic matter and R: relativistic matter) and decreases as the relativistic matter's energy density decreases (shown in fig. (4) for ω_{eff} and fig. (5) for relic densities). Initially, the relic density of non-relativistic matter increases while the relic density of relativistic matter decreases. The relic density of dark energy field approximately remains negligible in the initial stage of evolution. The time period when non-relativistic matter dominates with its relic density $\Omega_{NR} \approx 1$ is when the Universe decelerates at the highest rate as non-relativistic matter domination pulls things inwards more than the outwards Higgs negative pressure. When $\omega_{eff} = 0$ the weighted negative pressure of dark energy fields and positive (inwards) pressure of non-relativistic matter cancel each other. After that ω_{eff} starts to decrease as the non-relativistic energy density decreases and the Higgs relic energy density increases (shown in fig. (5)). From this time on the Higgs negative pressure dominates and ω_{eff} eventually settles down to -1. Note that the initial conditions for the charged field took the dark energy (vacuum) not to be charged.

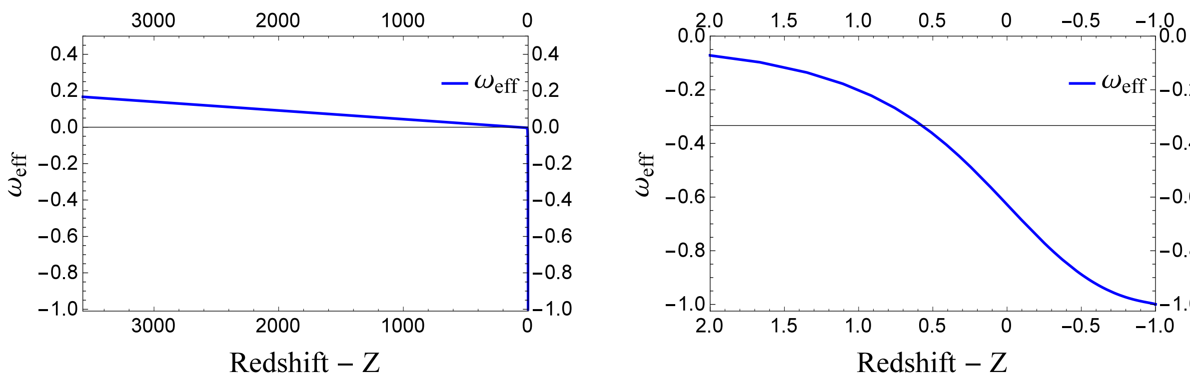

Figure 4. Effective equation of state parameter $\omega_{eff} = \Omega_{DE}\omega_{DE} + \Omega_R\omega_R + \Omega_M\omega_M$.

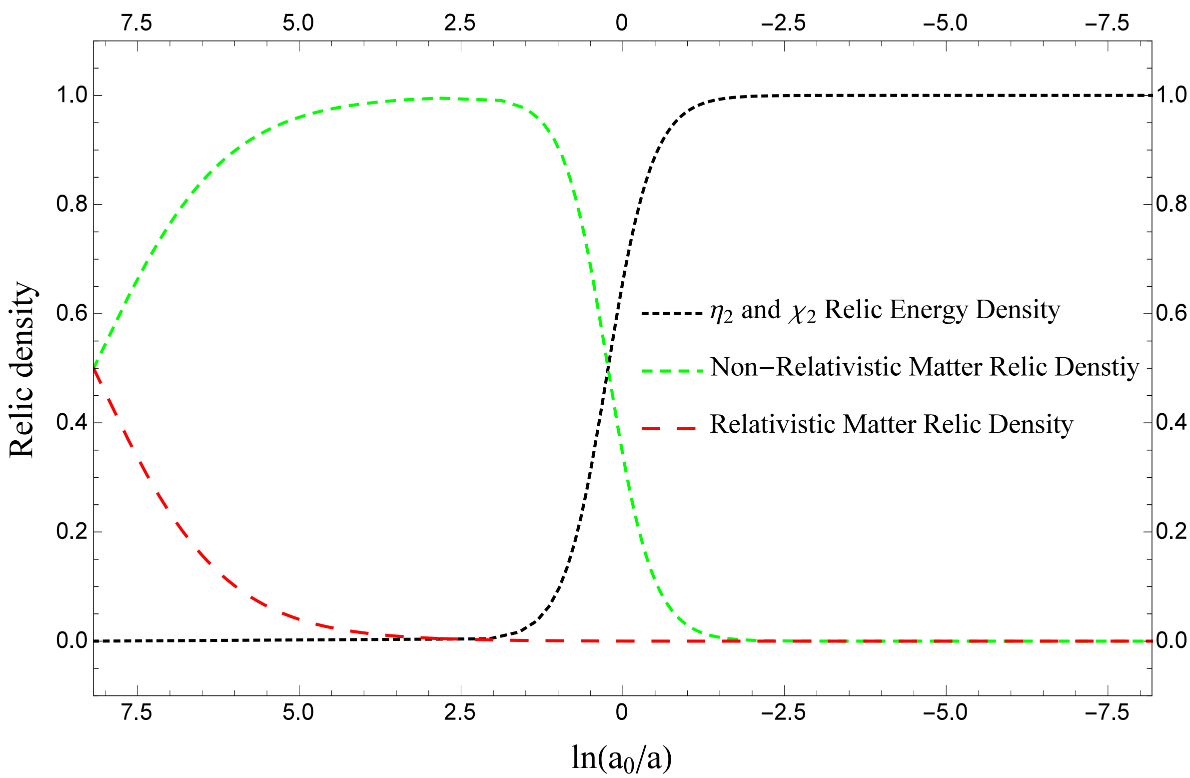

Figure 5. Relic densities of different components as a function of time.

Decay(s) of Higgs field(s)

At the tree level the doublet ϕ_2 in inert UDW-2HDM discussed above (whose component fields are the candidate for dark energy) is only coupled to the doublet ϕ_1 which acts in an identical way as SM Higgs field.

The interaction Lagrangian of Higgs fields of doublet ϕ_2 with Higgs fields of doublet ϕ_1 and gauge bosons (extracted from eq. (11)) is

$$
\begin{aligned}
\mathscr{L}_I \;=\;& \frac{v}{2}\eta_1{\eta_2}^2(\lambda_3+\lambda_4+\lambda_5)+\frac{v}{2}\eta_1{\chi_2}^2(\lambda_3+\lambda_4-\lambda_5)+v\lambda_3\eta_1\phi_2^{c\,2}+\frac{1}{4}{\eta_1}^2{\eta_2}^2(\lambda_3+\lambda_4+\lambda_5)\\
&+\frac{1}{4}{\eta_1}^2{\chi_2}^2(\lambda_3+\lambda_4-\lambda_5)+\frac{1}{2}{\eta_1}^2\phi_2^{c\,2}\lambda_3+\frac{1}{4}{\eta_2}^2{\chi_2}^2\left(m_{22}^4\Lambda_2^2\rho_2+\lambda_2\Lambda_4\rho_4\right)\\
&+\frac{1}{2}{\eta_2}^2\phi_2^{c\,2}\left(m_{22}^4\Lambda_2^2\rho_2+\lambda_2\Lambda_4\rho_4\right)+\frac{1}{2}{\chi_2}^2\phi_2^{c\,2}\left(m_{22}^4\Lambda_2^2\rho_2+\lambda_2\Lambda_4\rho_4\right)\\
&+\frac{g_2^2{g'_2}^2}{({g'_2}^2+g_2^2)}\phi_2^+\phi_2^-A_\mu^2+\frac{({g'_2}^2+g_2^2)}{8}\eta_2^2Z_\mu^2+\frac{({g'_2}^2+g_2^2)}{8}\chi_2^2Z_\mu^2+\frac{(g_2^2-{g'_2}^2)^2}{4({g'_2}^2+g_2^2)}\phi_2^+\phi_2^-Z_\mu^2\\
&+\frac{g_2g'_2(g_2^2-{g'_2}^2)}{({g'_2}^2+g_2^2)}\phi_2^+\phi_2^-A_\mu Z_\mu+\frac{g_2^2}{4}W^-{}_\mu W^+{}_\mu(\eta_2^2+\chi_2^2+2\phi_2^+\phi_2^-)\\
&+\frac{{g_2}^2g'_2}{2\sqrt{({g'_2}^2+g_2^2)}}\eta_2A_\mu(\phi_2^+W^-{}_\mu+\phi_2^-W^+{}_\mu)+i\frac{{g_2}^2g'_2}{2\sqrt{({g'_2}^2+g_2^2)}}\chi_2A_\mu(\phi_2^-W^+{}_\mu-\phi_2^+W^-{}_\mu)\\
&-\frac{g_2{g'_2}^2}{2\sqrt{({g'_2}^2+g_2^2)}}\eta_2Z_\mu(\phi_2^+W^-{}_\mu+\phi_2^-W^+{}_\mu)-i\frac{g_2{g'_2}^2}{2\sqrt{({g'_2}^2+g_2^2)}}\chi_2Z_\mu(\phi_2^-W^+{}_\mu-\phi_2^+W^-{}_\mu).
\end{aligned}
\tag{33}
$$

To suppress the interaction of Higgs fields, η_2, χ_2 and $\phi_2^\pm$, with the gauge bosons the idea is that the SU(2) doublet ϕ_2 is very weakly (different than ϕ_1) coupled with the gauge bosons, thus we have $g_2 \ll g_1$ and $g'_2 \ll g'_1$.

The decay width of the Higgs to a pair of Higgs scalars, using only the on-shell width, is given by[25,26]

$$
\Gamma(H_i \longrightarrow H_jH_k)=(2-\delta_{jk})m_{H_i}\frac{|C_{H_iH_jH_k}|^2}{32\pi}\sqrt{f\left(1,\frac{m_{H_j}^2}{m_{H_i}^2},\frac{m_{H_k}^2}{m_{H_i}^2}\right)}\;, \tag{34}
$$

where

$$
f\left(1,\frac{m_{H_j}^2}{m_{H_i}^2},\frac{m_{H_k}^2}{m_{H_i}^2}\right)=\left(1-\frac{m_{H_j}^2}{m_{H_i}^2}-\frac{m_{H_k}^2}{m_{H_i}^2}\right)^2-4\,\frac{m_{H_j}^2}{m_{H_i}^2}\,\frac{m_{H_k}^2}{m_{H_i}^2}\;,
$$

and $C_{H_iH_jH_k}$ is the coupling of different Higgs bosons H_i, H_j and H_k.

From eq. (33), $C_{\eta_1\chi_2\chi_2}=\frac{1}{2}v(\lambda_3+\lambda_4-\lambda_5)$, $C_{\eta_1\eta_2\eta_2}=\frac{1}{2}v(\lambda_3+\lambda_4+\lambda_5)=$ and $C_{\eta_1\phi_2^+\phi_2^-}=\lambda_3 v$.

The decay rates of η_1 to $\eta_2\eta_2$, $\chi_2\chi_2$, $\phi_2^-\phi_2^+$ for the masses used in the cosmological evolution determination are zero. The decay rates of the SM-like Higgs boson are given in the graph below as a function of masses m_{η_2}, m_{χ_2}, $m_{\phi_2^\pm}$ (on one axis) and their (η_2, χ_2, $\phi_2^\pm$) couplings with the SM- like Higgs (on the other axis),

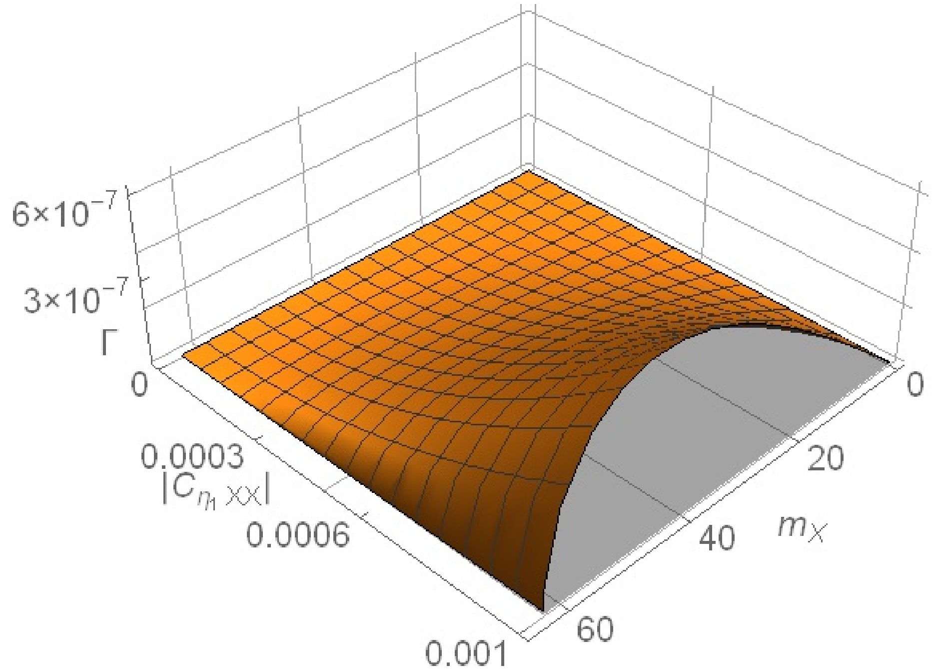

Figure 6. Decay rates for $\eta_1 \longrightarrow$ xx.

It should also be mentioned that the total decay width of the SM-like Higgs boson here is well within the bounds of mass resolution $\approx 12 \times 10^{-3}$GeV of LHC[28] for the values of the masses used in the cosmological evolution of the Higgs fields. One should also mention that the SM prediction of total decay width for the Higgs boson is 4.21×10^{-3}GeV with mass 126GeV[29] and is 4.07×10^{-3}GeV with mass 125GeV[28]. In this model, we get three more decay channels of the SM-like Higgs, to the other Higgs bosons pair.

Conclusion and Discussion

Scalar fields are among the possible candidates for the observed accelerated expansion of the Universe. In this article, we have argued that the Particle Physics developed so far must have something in, or minimally beyond, the SM which will explain the observed accelerated expansion of the Universe and hence will serve as a dark energy candidate. Here we assumed that dark energy is actually some scalar field which is present as the Higgs in a model where the potential has the non-degenerate vacua, we called this model uplifted double well two-Higgs doublet model (UDW-2HDM).

We found that if the present Universe is described by the true vacua of UDW-2HDM then the component fields of the second doublet ϕ_2 (which acts as the inert doublet) can be one possible candidate for the dark energy. As the present contribution of the dark energy to the critical energy density is about 0.7, this value is obtained by taking the mass of the CP-even field's mass small ($O(10^{-59})$GeV). The most important thing is that with the initial conditions set, the mass of the charged ($\phi_2^{\pm}$) field becomes arbitrary. Hence this model will fit for any value of mass of $\phi_2^{\pm}$. One also needs to keep in mind that the values of masses were chosen arbitrarily so as to get dark energy relic density ≈ 0.7. Changing the values of the masses, the relic density does not change much.

It should also be mentioned that if we remove the Z_2 symmetry, the second Higgs doublet does not remain inert. Thus in the case of Z_2 violation (soft or hard), the CP-even Higgs fields will mix by an angle β. In that case a new parameter (β) will arise in the theory. Obtaining a dark energy candidate in that model will require fine tuning in the Yukawa interactions in such a way that either the dark energy field does not couple or couple very weakly with the fermions.

Since the inflation and late time acceleration (from a dynamical field) are not very different, one usually talks about the inflaton field decay at the end of inflation into another (dark) field called dark radiation (since it does not couple to SM that is why it is called dark radiation) which may then govern the interaction between dark matter and normal matter or couple to the dark matter. If this dark radiation is a massive Planck-coupled particle which exist post-inflation, then these particles will inevitably dominate the universe energy density of the Unvierse later. Since these particles will dominate the energy density of the Unvierse, they can be possible candidate of current accelerated expansion of the Universe. The interesting fact about this dark radiation is that it decouples from the SM sector at the end of inflation. There are quite a few observation that this dark radiation could actually exist[30–33]. This fact introduces stringent constraints on models involving the dark radiation. The usual talked dark radiation field is the sterile neutrino. In these models, one tries to investigate the effective number of neutrino species N_{eff} where they rule in or rule out $N_{eff} \approx 4$. In our case here, we do not intend to present any model of inflation nor dark radiation. But if our Higgs field gets coupled to the dark matter after inflation this could introduce constraints on our model. This should be investigated in the future.

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Author contributions statement

Muhammad Usman as the first author, has done all the research work presented. Muhammad Usman is also responsible for the development of the model. The suggestions were given by Asghar Qadir to improve the work along with the supervision. Both authors have carefully proofread the manuscript.

Additional information

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