A Five-Dimensional Approach to Dark Matter and Dark Energy

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A naive five-dimensional model simultaneously explains dark energy and the flat rotation curves of galaxies and enhanced gravitational lensing usually attributed to dark matter. An additional (fifth) dimension is invoked which corresponds to the radius of curvature of four-dimensional space-time and does not represent a degree of freedom of motion. The universe so modelled has two modes of expansion: that of three-dimensional space and that of the fifth dimension itself, where the latter has the characteristics partly of space and partly of time. The boundary between four-dimensional space-time and the fifth dimension is modelled by contours along which energy is conserved. The model is able to reproduce: (i) the observed relationship between distance modulus and red shift for Type 1a supernovae; (ii) the Tully-Fisher relationship; and (iii) gravitational lensing effects beyond general relativity for galaxies and clusters of galaxies.

Keywords: dark matter, dark energy, additional dimensions

1. Synposis

A discrepancy exists between the observed extent of gravitating matter in the universe and that inferred from galaxy rotation curves, galaxy cluster dynamics, gravitational lensing and simulations of evolution of the universe. The existence of abundant unobserved (or dark) matter has been proposed by many cosmologists to account for this discrepancy, although to date dark matter has not been directly detected.

Observations concerning the luminosity distance of Type 1a super-novae, their red shift and the rate of expected Hubble expansion of the universe have led to the conclusion that the expansion has been accelerating since about six billion years ago. This has led many to propose that dark energy permeates the universe, counteracting the decelerating effect of gravity. However, to date, the source of dark energy has not been identified.

It is the purpose of this presentation to show that a universe comprised of five rather than four large-scale dimensions and without the assumed presence of dark matter and dark energy, could account for these same observations concerning galaxies, clusters of galaxies and the expansion rate of the universe. The possibility of additional dimensions of space-time has, of course, long been under discussion within theoretical physics. However, the distinctive features of the dimension considered in this presentation include its large scale and its hybrid character, being in partly like space and partly like time.

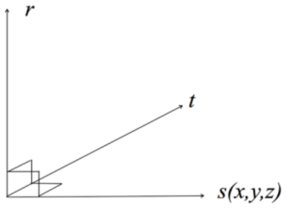


Fig 1. Introducing a fifth dimension (r)

We consider a naive model universe in which three-dimensional space *s(x,y,z)* is isotropic, closed and expanding. The radius of curvature in this universe is given the symbol *(r)* and dimensions of length and is defined to be orthogonal to the three space dimensions *(s)* and time *(t)* (Fig 1). If Newton's Second Law can be extended to five dimensions, then energy propagating through three-dimensional space that is closed and forming the “surface volume" of a four-dimensional hyper-sphere of radius *r*, will experience a virtual acceleration a_r (1) acting in the direction of *r* increasing. It is assumed to be fundamental that energy is conserved within four-dimensional space-time, notwithstanding that it is embedded within five dimensions.

$$a_r = \frac{c^2}{r} \tag{1}$$

An axial vector *(Ω)* representing angular velocity is aligned normal to the plane of rotation xy of the rotating body it describes. However, with an additional dimension, *r*, the direction of *Ω* can lie anywhere in the zr plane, so more information is required to define uniquely the orientation of *Ω* within a five dimensional framework.

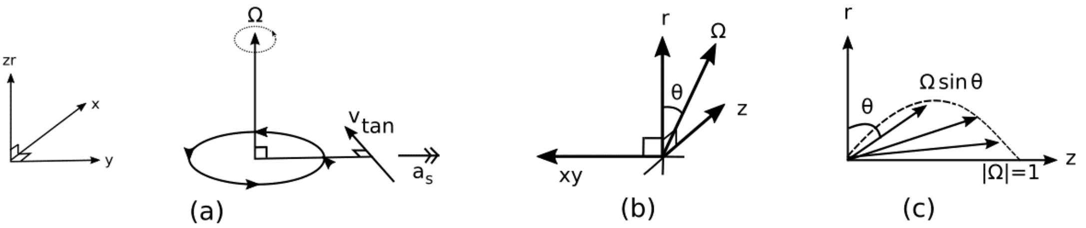


Fig 2. Orientation of angular velocity vectors in five dimensions

This additional information comes from including the influence of the additional acceleration a_r on the body which defines the orientation θ of the angular velocity vector *Ω* in the zr plane (Fig 2). This analysis ultimately leads to a derivation of the Tully-Fisher relationship and so to the prediction of flat rotation curves for galaxies, without the need for dark matter to be assumed. This analysis does not predict any departure from the expected four-dimensional Newtonian behaviour for stars at the fringe of galaxies as regards their motion normal to their planes of rotation, or radially from the centre of the galaxy.

Remote from gravitating matter, the conservation of energy at the boundary between four-dimensional space-time and the fifth dimension can be defined in terms of the fifth dimensional parameter *r* (2):

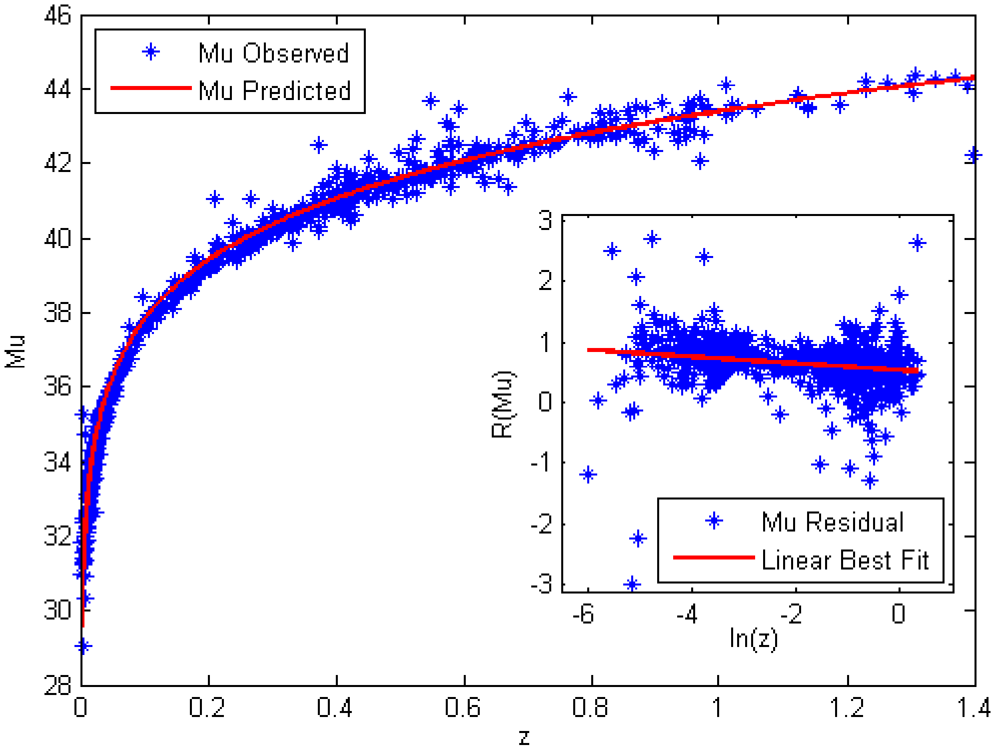
$$\frac{c^2}{r} + \ddot{r} = 0 \tag{2}$$

When describing the gravitational deflection of light beams passing close to galaxies or galaxy clusters, the background expansion of the universe ($\dot{r}$) in the direction of the fifth dimension must be taken into account. That is, the extent of gravitational lensing observed for galaxies and galaxy clusters, which is stronger than that implied by baryoninc matter alone, appears in the analysis to be due to the background rate of expansion of the universe (in the fifth dimension) at the fringes of these systems. Lensing multiples of the order 8-13 are predicted by a five dimensional model without the need for dark matter to be assumed (Fig 3).

	Baryonic Mass (Kg)	Impact Parameter (S) (m)	Gravitational acceleration (ms^{-2})	dr/ds	Lensing Multiple ($4GM/c^2S$)
Sun	2×10^{30}	7×10^{8}	272	$\sim 10^{12}$	1
Galaxy	8×10^{41}	1×10^{21}	5.3×10^{-11}	0.3 – 0.4	~ 8 – 13
Cluster	1×10^{45}	1.4×10^{23}	3.4×10^{-12}	0.02 - 0.03	~ 8 -13

Fig 3. Gravitational Lensing Results

A universe defined in five dimensions rather than four has two modes of expansion: the first concerns the expansion of (three dimensional) space at a constant value of the fifth dimensional parameter *r*; and the second concerns the expansion of (three dimensional) space due to increases in the parameter *r* itself. Accordingly, Hubble's constant can be decomposed into two terms, referred to respectively as H3 and H5, whose values contribute to a Hubble constant (for the current era) of 68.1. kms^{-1} Mpc^{-1}, as follows: H3 = 66.5 kms^{-1} Mpc^{-1} and H5 = 1.6 kms^{-1} Mpc^{-1}. The competing effects of these two modes of expansion can give rise to either an accelerating or decelerating expansion of the universe.



Fug 4. Type 1a SN luminosity distance modulus (μ) vs red shift (z) comparison between observation and five-dimensional model predictions

The naive five dimensional model has been used to fit the available data of luminosity distance and red shift for Type 1a super-novae (http://supernova.lbl.gov/Union/Union2.html) (Fig 4). The model does not indicate the presence of an unaccounted accelerating expansion of the universe, which has been taken to indicate the presence of dark energy.

Two principles underpin the analysis undertaken: first, the extension of Newtonian equations of motion from four into five dimensions; and second, the conservation of energy at the boundary between four-dimensional space-time and a fifth dimension. The fifth dimension introduced is large in scale: the value suggested by the analysis lies in the range 4.8 - 7.7X10^{26} m.

The balance between accelerations directed in three-dimensional space and those directed in the fifth dimension determines the gradient of contours *r(s)* which define the boundary between four-dimensional space-time and the fifth dimension. This condition of balance (or equilibrium) between several accelerations can alternatively be thought of as defining a “surface tension" within the contour at the boundary between four-dimensional space-time the fifth dimension, which in (*x,y,z,r*) dimensions become branes. This approach to the determination of the local curvature of space-time suggests a more than coincidental basis for the rubber sheet analogy of general relativity.

As the gradient (*dr/ds*) of an *r(s)* contour reduces towards (and drops below) unity, its influence on the local orientation of angular velocity vectors has to be taken into account when determining orbital motion within four dimensions. These (*dr/ds*) gradients arise when the acceleration directed in three-dimensional space approaches (or drops below) the local value for the acceleration directed in the fifth dimension (a_r) which has a background value in the current era suggested by the analysis of 1.2 - 1.9X10^{-10} ms^{-2} (which incorporates the value reported by Milgrom for the MOND constant). The derivation of the contour gradients (*dr/ds*) can be shown to be consistent with special and general relativity.

2. Conclusion

If five-dimensional effects are modelled, then predictions can be made for orbital dynamics within galaxies, the enhanced gravitational lensing of galaxies and galaxy clusters, and the relationship between distance modulus and red shift for Type 1a SN, which are consistent with observations without the need to introduce dark matter or dark energy.

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