Self–gravitating particles, entropy, and structure formation

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Note: this title and abstract supersedes the version in the online "Talk Detail".

I outline a kinetic theory model of gravitational collapse due to a small perturbation. This model produces a pattern of entropy destruction in a spherical core around the perturbation, and entropy creation in a surrounding halo. Core–halo patterns are ubiquitous in the astrophysics of gravitational collapse, and are found here without any of the prior assumptions of such a pattern usually made in analytical models. Motivated by this analysis, I outline a possible scheme for identifying structure formation via data from observations or a simulation. This might aid exploration of hierarchical structure formation, supplementing the usual density–based methods for highlighting astrophysical and cosmological structure at various scales.

Keywords: Kinetic theory; Gravitational collapse; Entropy; Structure formation.

1. Introduction and Motivation

Many astrophysical contexts see gravitational collapse leading to structure formation. A simple model of gravitational collapse can be constructed based on the virial theorem, with an artificial division between a central core and surrounding halo.[1] Entropy rises within the halo, with at least a relative entropy decrease in the shrinking core. If we further assume that the core's density profile scales with its radius R, then its phase space volume varies like $R^{3/2}$ and it is easily seen that there is an absolute fall in entropy within the core.

Note that there may, at least in principle, be two distinguishable entropy–related effects: entropy transport from one volume element to another; and entropy creation/destruction. The distinction is somewhat arbitrary in the artificial virial theorem model just discussed. In kinetic theory treatments however, entropy creation is separately identifiable, and arises only from collisional effects.

This note is based on a full account in Ref. 2. I outline how to construct a kinetic theory model of gravitational collapse allowing analytical description of entropy creation (Sec. 2). A core–halo pattern emerges as a result (Sec. 3). This suggests an approach to identifying structure in simulations and observations (Sec. 4).

2. Outline of the Model

The model consists of a small central perturbation to an underlying uniform distribution of self–gravitating particles. Via the well–known "Jeans swindle," a system with a finite number of particles N in a bounded volume can be used to model a uniform arrangement of particles which is unbounded in extent, and so stable if unperturbed.[1]

The perturbation evolves under truncated first order BBGKY equations for the evolution of the distribution function (DF) and the correlation function. Write

f_0, f_1 for the underlying and perturbation DFs, similarly g_0, g_1 for the correlation functions, and set $\mathbf{a}_{1,2}$ to be N times the acceleration of a particle at $1 = (\mathbf{x}_1, \mathbf{v}_1)$ due to a particle at $2 = (\mathbf{x}_2, \mathbf{v}_2)$. The full first order equation for the DF's evolution is[3]

$$\frac{\partial f(1)}{\partial t} + \mathbf{v}_1 \cdot \frac{\partial f_1(1)}{\partial \mathbf{x}_1} + \int \mathbf{a}_{1,2}\, f_1(2)\, \mathrm{d}(2) \cdot \frac{\partial f_0(1)}{\partial \mathbf{v}_1} = -\frac{1}{N} \frac{\partial}{\partial \mathbf{v}_1} \int \mathbf{a}_{1,2}\, g_1(1,2)\, \mathrm{d}(2)\,, \quad (1)$$

where the right–hand side gives the "collisional" effects of two–body interactions.

Both f_0 and f_1 are assumed to have initial Maxwellian velocity distributions, with parameter σ, and a typical scale for the system is then given[1] by the "Jeans wave number," $k_\mathrm{J} = \sqrt{4\pi G m n}/\sigma$, where m is the (identical) mass of each particle, n is the average number density, and G is Newton's gravitational constant. The model is valid for the beginning of gravitational collapse — the initial period during which the perturbation remains small.

The entropy creation rate comes entirely from collisional effects, and its density at $\mathbf{x}_1$ is

$$\left(\frac{\partial S_{\mathbf{x}_1}}{\partial t}\right)_{\text{creation}} = \int \ln\left[f(1)\right] \frac{\partial}{\partial \mathbf{v}_1} \cdot \int \mathbf{a}_{1,2}\, g(1,2)\, \mathrm{d}(2)\, \mathrm{d}^3\mathbf{v}_1, \quad (2)$$

where $f = f_0 + f_1$ and $g = g_0 + g_1$. It is useful to coarse grain the entropy, both because it enables progress to be made with analytical calculations, and because identification of structure typically implies focusing on only a range of scales.

The approach of Ref. 2 coarse grains by choosing a parameter $\beta \ll 1$, and then in applying Eq. (2) considering only wave numbers $k < \beta k_\mathrm{J}$. Furthermore, Ref. 2 focuses on only the "asymptotically dominant" part of the DF, correlation, and entropy creation — the part that soon comes to grow fastest.

3. Result: a Core–Halo Pattern

I briefly describe the approach of Ref. 2 to calculating the resulting asymptotically–dominant coarse–grained (acg) entropy creation rate, and in particular its dependence on the distance r from the initial central perturbation. Let S°_acg be the acg entropy created within a sphere of radius r in the time t since the initial perturbation was introduced. For large N, Eq. (1)'s collisional term is highly suppressed, and, at leading order in $1/N$, we can ignore it in calculating the contribution of the DF to the first order correlation equation. The resulting equations can then be solved to give, to leading order in $1/N$, perturbation size ϵ, and β,

$$\frac{\partial^2 S^\circ_\mathrm{acg}}{\partial t\, \partial r} = \frac{2k_\mathrm{J}^2\, \sigma\, \mathrm{e}^{3k_\mathrm{J}\sigma t}}{9\pi^2 \beta} \left(\frac{\epsilon N}{nB}\right)^2 \hat{S}^\circ_\mathrm{acg}(k_\mathrm{J}\beta r)\,, \quad (3)$$

with $B = 4\pi/(k_\mathrm{J}\beta)^3$ being the volume of a sphere associated with the coarse–graining scale, and $\hat{S}^\circ_\mathrm{acg}$ is a function shown in Fig. 1. Note that Eq. (3) shows the entropy creation rate's density on a shell of radius r around the central perturbation.

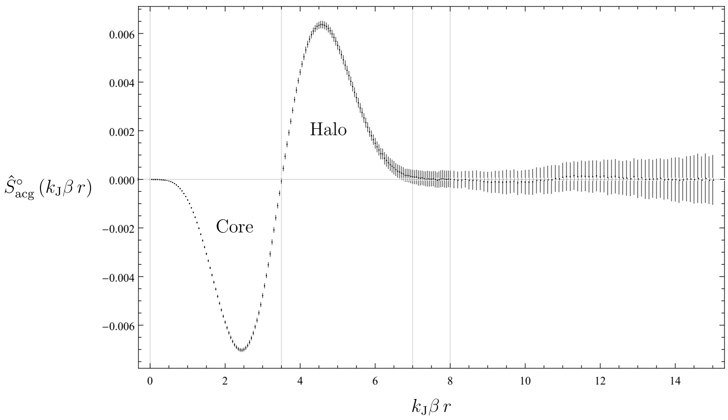

Fig. 1. The entropy–creation pattern function S°_{acg}, calculated by numerical integration of an analytical expression. The error bars show integration error estimates. Adapted from fig. 2 of A core–halo pattern of entropy creation in gravitational collapse, A. J. Wren, MNRAS, **477** (2018).

Entropy destruction occurs in a "core" around the central perturbation, with, at leading order, equal[2] and opposite entropy creation in a "halo" extending for a finite radius beyond that core, as shown in Fig. 1. The physical scale for the core–halo pattern depends on the coarse–graining parameter β: the coarser the graining, the bigger the pattern's physical scale. Entropy destruction (resp. creation) corresponds to collisional relaxation (resp. "de–relaxation") of the perturbation.

A core–halo pattern of gravitational collapse, well known from simulations and observations, is generally set "by hand" in analytical models. As far as the author has been able to determine, this is the first time an analytical kinetic theory model has produced a core–halo pattern.

4. Structure Formation in Simulations and Observations

It is well known that the Universe has a multi–scale hierarchical structure, in which core–halo patterns are ubiquitous. The identification of observed or simulated astrophysical structure typically involves considering features of especially high or low densities, in physical space, or phase space. There is no unambiguous definition of structure in this context, which can result in different methods giving different results — for example, in identifying sub–haloes near the centre of dark matter haloes,[4] in major halo mergers,[5] and in classifying elements of the cosmic web.[6] This suggests that complementary methods for identifying structure, or structure formation, may be helpful.

Given data from observations or a simulation, the above analysis suggests we might construct a coarse–grained particle DF and correlation function, which then could give the entropy creation rate from Eq. (2). The pattern of coarse–grained entropy creation might then give a way to identify structure formation, presumably associated with volumes of relatively lower entropy creation.

A key step is extracting a correlation function from the data. There are various methods for doing this.[7] At least in the kinetic theory model of Sec. 2, the correlation between two particles is small compared with their joint DF. In calculating correlation functions from data, this would imply[7] a need to identify the difference between two quantities of relatively similar size.

This means that robust identification of entropy creation, and hence structure formation, may need rather precise data. The outlook for data of sufficient precision is perhaps encouraging with the development of ever more powerful computer simulations, and the availability of detailed phase space observations from, for example, *Gaia*.[8]

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