Warped 5D Cosmic Strings, Conformal Invariance and the Quasar Link

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Topological defects formed in the early stages of our universe can play a crucial role in understanding anisotropic deviations of the Friedmann Lemaître Robertson Walker model we observe today. These defects are the result of phase transitions associated with spontaneous symmetry breaking in gauge theories at the grand unification energy scale. The most interesting defects are cosmic strings, vortex-like structures in the famous gauged U(1) abelian Higgs model with a 'Mexican-hat" potential. Other defects, such as domain walls and monopoles are probably ruled out, because they should dominate otherwise the energy density of our universe. This local gauge model is the fundament of the standard model of particle physics, where the Higgs-mechanism provides elementary particles with mass. It cannot be a coincidence that this model also explains the theory of superconductivity. The decay of the high multiplicity (n) super-conducting vortex into a lattice of n vortices of unit magnetic flux is energetically favourable and is experimentally confirmed. It explains the famous Meissner effect. This process could play a fundamental role by the entanglement of cosmic strings just after the symmetry breaking. The stability of the lattice depends critically on the parameters of the model, especially when gravity comes into play. The questions is how the imprint of the cosmic strings could be observed at present time. Up to now, no evidence is found. The recently found alignment of the spinning axes of quasars in large quasar groups on Mpc scales, could be a first indication of the existence of these cosmic strings. The temporarily broken axial symmetry will leave an imprint of a preferred azimuthal-angle on the lattice. This effect is only viable when a scaling factor is introduced. This can be realized in a warped five dimensional model. The warp factor plays the role of a dilaton field on an equal footing with the Higgs field. The resulting field equations can be obtained from a conformal invariant model. Conformal invariance, the missing symmetry in general relativity, will then spontaneously be broken, just as the Higgs field. The dilaton field, or equivalently, the warp factor, could also contribute to the expansion of the universe as it can act as a dark energy term coming from the bulk spacetime. It makes the cosmic string temporarily super-massive. This process could solve the cosmological constant and hierarchy problem. It is conjectured that the dilaton field has a dual meaning. At very early times, when the dilaton field approaches zero, it describes the small-distance limit of the model, while at later times it is a warp (or scale) factor that determines the dynamical evolution of the universe. When more data of quasars of high redshift will become available, one could proof that the alignment emerged after the symmetry breaking scale and must have a cosmological origin. The effect of the warp factor on the second-order perturbations could also be an indication of the existence of large extra dimensions.

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I. INTRODUCTION

The standard model (SM) of the electroweak and strong interactions is a successful framework in which one studies elementary particles and includes the principles of quantum mechanics (QM). On the other hand, general relativity (GR) is also a very impressive theory constructed by theoretical physicists. It describes large scale structures in our universe and one can construct solutions which are related to real physical objects, for example the Kerr solution, the end stage of a collapsing spinning star. A legitimate question is if there are other axially symmetric solutions in GR. It came as a big surprise that there exist vortex-like solutions in Einstein's theory. These vortex solutions occur as topological defects at the symmetry breaking scale in the Einstein-abelian U(1) scalar-gauge model, where the gauge field is coupled to a complex charged scalar field[1–4]. The solution shows a surprising resemblance with type II superconductivity of the Ginzburg-Landau(GL) theory[5, 6], where the electro-magnetic(EM) gauge invariance is broken and the well-known Meissner effect occurs[7, 8]. One says that the phase symmetry is spontaneously broken and the EM field acquires a length scale, which introduces a penetration depth of the gauge field A_μ in the superconductor and a coherence length of Φ. In the relativistic case one says that the photon acquires mass. Because we have three space dimensions, these solutions of the GL theory behave like magnetic flux vortices (Nielsen Olesen strings[3]) extended to tubes and carry a quantized magnetic flux $2\pi n$, with n an integer, the topological charge or winding number of the field. It was discovered by Abrikosov[8] that these vortices can form a lattice. These localized vortices (or solitons) in the GL-theory are observed in experiments. The phenomenon of magnetic flux quantization in the theory of superconductivity is characteristic for so-called ordered media. The vortex solution possesses mass, so it will couple to gravity. The resulting self-gravitating cosmic strings (CS) still show all the features of superconductivity, but the stability conditions complicate considerably. The stability of the formed lattices depends critically on the parameters of the model, certainly when gravity comes into play. The force between the gauged vortices depends on the ratio $\alpha \equiv \frac{m_A^2}{m_\Phi^2}$, i.e., the masses of the gauge and scalar field, the GL parameter, the energy scale at which the phase transition takes place and the spacetime structure. When the mass of the Higgs field is greater than the mass of the gauge field, vortices will repel each other. So gravity could balance the vortices. The energy of the vortex grows by increasing multiplicity n, so configurations with $n > 1$ can be seen as multi-soliton states and it is energetically favourable for these to decay into n well separated $n = 1$ solitons. Vortices with high multiplicity can be formed during the symmetry breaking. The total vortex number n is the sum of multiplicities $n_1, n_2, ..$ of isolated points (zero's of Φ)[6].

Our universe, described by a spatially homogeneous and isotropic Friedmann Lemaître Robertson Walker (FLRW) spacetime, shows significant large-scale inhomogeneous structures, for example, the cosmic web of voids with galaxies and clusters in sheets, filaments and knots, the angular distribution in the cosmic microwave background (CMB) radiation and the recently found alignment of polarization axes of quasars in large quasar groups(LQG's) on Mpc-scales[9, 10]. The question is if these complex nonlinear structures of deviation from isotropy and homogeneity have a cosmological origin at a moment in the early stage of the universe. One possibility of this origin could be a CS-network formed by the self-gravitating Einstein-scalar-gauge model. A pleasant fact is that this model has very few parameters and hence more appealing than other models such as inflationary models. It is believed that the mass per unit length of the CS is of the order of the GUT scale, $G\mu \approx 10^{-7}$. Observational bounds, however, predict a negligible contribution of CS's to initial density perturbation from which galaxies and clusters grew. Besides the inconsistencies with the power spectrum of the CMB, radiative effects of the CS embedded in a FLRW spacetime are rapidly damped in any physical regime[11]. Further, the lensing effect of these CS's are not found yet.

There is, however, another possibility to detect the presence of CS's. On a warped spacetime, the fields can become temporarily super-massive by the warp factor in the framework of string theory (or M-theory). Naively one expect that gravity will play a subordinate role compared with the other fields. In 4D counterpart models this is true, but not in warped spacetimes. The super-massive CS's can be formed at a symmetry breaking scale much higher than the GUT scale, i.e., $G\mu >> 1$. So their gravitational impact increases considerably, because the CS builds up a huge mass in the bulk space. Here we consider the warped brane world model of Randall-Sundrum (RS)[12, 13], with one large extra dimension. The result is that effective 4D Kaluza-Klein(KK) modes are obtained from the perturbative 5D graviton. These KK modes will be massive from the brane viewpoint. The modified Einstein equations on the brane and scalar gauge field equations will now contain contributions from the 5D Weyl tensor[14–17]. In order to explore these effective field equations, we apply an approximation scheme, i.e., a multiple scale method(MSM). In this method one can handle the decay of the n-vortex in a perturbative way. The MSM or high-frequency method is an approved tool to handle nonlinearities and secular terms arising in the partial differential equations(PDE) in GR. When there is a high curvature situation, a linear approximation of the Einstein equations is not suitable[18–20].

Other issue related to our 5D warped spacetime is the behavior at small scales, i.e., when the warp factor or scale factor of the spacetime becomes very small. We conjecture that our warp factor becomes the dilaton field which is needed to make the Lagrangian conformal invariant. Breaking of the conformal symmetry (which will occur when other fields come into play; after all we experience today a huge discrepancy in scales), can be compared with the BEH mechanism in the standard model of particle physics.

In section 2 we will outline the model under consideration. This section is a review of a former study[21–24]. In section 3 we will explain the connection with conformal invariance.

II. THE SUPERCONDUCTING STRING MODEL IN WARPED SPACETIME

A. Outline of the model

Our model will be based on a warped five-dimensional FLRW spacetime

$$ds^2 = \mathcal{W}(t,r,y)^2\Big[e^{2(\gamma(t,r)-\psi(t,r))}(-dt^2+dr^2)+e^{2\psi(t,r)}dz^2+r^2e^{-2\psi(t,r)}d\varphi^2\Big]+dy^2, \tag{1}$$

with $\mathcal{W}=W_1(t,r)W_2(y)$ is the warp factor. All standard model fields reside on the brane, while gravity can propagate into the bulk. The 5D Einstein equations are[17]

$${}^{(5)}G_{\mu\nu}=-\Lambda_5{}^{(5)}g_{\mu\nu}+\kappa_5^2\delta(y)\Big(-\Lambda_4{}^{(4)}g_{\mu\nu}+{}^{(4)}T_{\mu\nu}\Big), \tag{2}$$

with $\kappa_5=8\pi{}^{(5)}G=8\pi/{}^{(5)}M_{pl}^3$, Λ_4 the brane tension, ${}^{(4)}g_{\mu\nu}={}^{(5)}g_{\mu\nu}-n_\mu n_\nu$, and n^μ the unit normal to the brane. The effective 4D Einstein-Higgs-gauge field equations are[17, 21]

$${}^{(4)}G_{\mu\nu}=-\Lambda_{eff}{}^{(4)}g_{\mu\nu}+\kappa_4^2{}^{(4)}T_{\mu\nu}+\kappa_5^4\mathcal{S}_{\mu\nu}-\mathcal{E}_{\mu\nu}, \tag{3}$$

$$D^\mu D_\mu\Phi=2\frac{dV}{d\Phi^*},\qquad {}^{(4)}\nabla^\mu F_{\nu\mu}=\frac{1}{2}i\epsilon\Big(\Phi(D_\nu\Phi)^*-\Phi^*D_\nu\Phi\Big), \tag{4}$$

with $D_\mu\Phi\equiv{}^{(4)}\nabla_\mu\Phi+i\epsilon A_\mu\Phi$, ${}^{(4)}\nabla_\mu$ the covariant derivative with respect to ${}^{(4)}g_{\mu\nu}$, $V(\Phi)=\frac{1}{8}\beta(\Phi\Phi^*-\eta^2)^2$ the potential of the Abelian Higgs model and η the symmetry breaking scale. $F_{\mu\nu}$ is the Maxwell tensor. The righthand side of the Einstein equations contains a contribution $\mathcal{E}_{\mu\nu}$ from the 5D Weyl tensor and carries information of the gravitational field outside the brane. The quadratic term in the energy-momentum tensor, $\mathcal{S}_{\mu\nu}$, arising from the extrinsic curvature terms in the projected Einstein tensor. ${}^{(4)}T_{\mu\nu}$ represents the matter content on the brane, in our case the scalar and gauge fields. It is clear that de general solution for the vortex will be cylindrical symmetric (polar coordinates $(r,z,\varphi$ and in the notation of NO), so we parameterize the self-gravitating scalar gauge field as

$$\Phi=\eta X(t,r)e^{in\varphi},\qquad A_\mu=\frac{n}{\epsilon}\big[P(t,r)-1\big]\nabla_\mu\varphi, \tag{5}$$

n is the topological charge or winding number of the scalar field. For a detailed treatment of the issue, we refer to Jaffe and Taub[5]. The warp factor can be solved from the 5D Einstein equations:

$$\mathcal{W}=W_2(y)W_1(t,r)=\frac{\pm e^{\sqrt{-\frac{1}{6}\Lambda_5}(y-y_0)}}{\sqrt{\tau r}}\sqrt{\Big(d_1e^{(\sqrt{2\tau})t}-d_2e^{-(\sqrt{2\tau})t}\Big)\Big(d_3e^{(\sqrt{2\tau})r}-d_4e^{-(\sqrt{2\tau})r}\Big)}, \tag{6}$$

In figure 1 we plotted several possible solutions. Note that the warp factor depends on r and t, so the contribution to the spacetime evolutions will be different for different stages in time. In the early universe the warp factor represents a

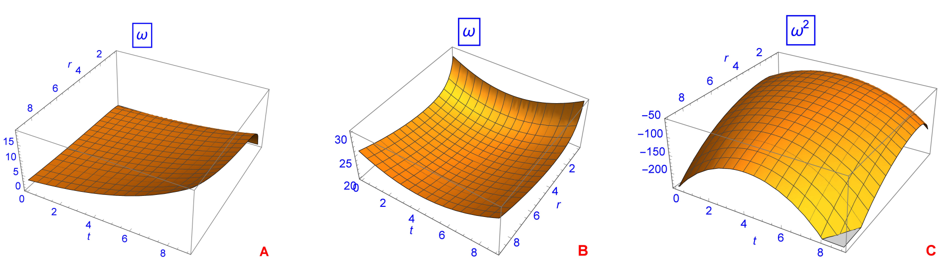

FIG. 1. Three different plots of the warp factor for some values of the constants d_i and τ.

dilaton field, conformally coupled to gravity. In section 3 we will return to this issue. The model under consideration is invariant under the group U(1) of local gauge transformations (of the second kind)

$$\Phi(\mathbf{x}) \to e^{i\chi(x)}\Phi(x), \qquad A_a(\mathbf{x}) \to A_a(x) + \frac{1}{e}\partial_a \chi(\mathbf{x}), \tag{7}$$

The conserved electromagnetic current becomes now $J^a = \frac{ie}{2}(\Phi D^a \Phi^* - \Phi^* D^a \Phi)$. Since the minima of $V(\Phi)$ are at $|\Phi| = \eta$, this symmetry is spontaneously broken and the field acquires a non-zero vacuum expectation value. Let us now take a closer look at the potential (figure 2). Suppose we take a closed loop L in physical space in polar coordinates(r, z, φ) around the string, where z is a kind of "dummy" coordinate: $\delta\varphi = 2\pi$. The far field will then take

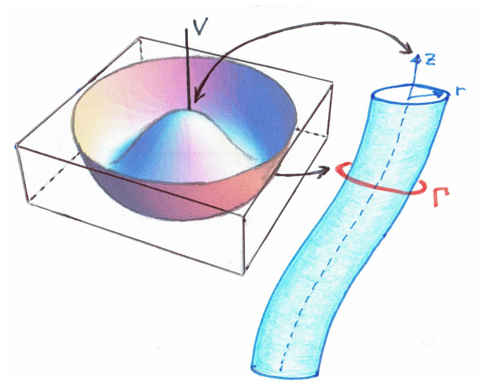

FIG. 2. **Mapping the degenerated minima of the potential to position space.**

the form $\Phi \approx \eta e^{i\varphi}$. The position of the string is located by taking the closed loop Γ. If we shrink Γ to a point, then the phase jump of 2π is no longer defined. This phase jump can only be resolved continuously if the field rises to the top of the potential $\Phi = 0$. We say that the energy of the "false" vacuum is trapped. This is a *topological defect*. The vacuum manifold $\mathcal{M}$ is not simply connected. $\mathcal{M}$ contains enclosed holes about which loops can be trapped. In a non-cylindrical symmetric configurations The curve Γ would shrink to a point and we produce a discontinuity in the phase factor, contradicting the smoothness of the Higgs field. The static finite energy configuration cannot be stable, since we can press it down to the vacuum. So in this case the jump is not allowed. However, the abelian Higgs model is topological stable by the cylindrical symmetry. In this physical cylindrical symmetric space we can have again similar non-trivial windings n about a degenerated circle of minima. By calculating the magnetic flux ($\frac{q}{\hbar} = e$)

$$\Theta = \oint_\Gamma \vec{A}.d\vec{r} = \frac{\hbar}{q}\oint_\Gamma \vec{\nabla}\phi.d\vec{r}, \tag{8}$$

and using Stokes's theorem, we obtain that the magnetic flux lines are quantized by $\frac{2\pi n}{e}$. One could wonder what happens when quantum fluctuations excites the vortex. This will be treated in the next section. The energy of the string in flat spacetime is given by

$$E = \frac{1}{2}\frac{n^2}{e^2 r^2}(\partial_r P)^2 + \frac{1}{2}\eta^2(\partial_r X)^2 + \frac{1}{2}n^2\eta^2\frac{P^2 X^2}{r^2} + \frac{1}{8}\beta\eta^4(X^2-1)^2 \tag{9}$$

The energy is proportional with n^2, so there can be no exact ground state for the string carrying multiple flux quanta (the expression changes when gravity comes into play and new features will emerge). There are some characteristic parameters:

$$penetration\ and\ coherence\ lenght: \nu = \frac{1}{e\eta}, \quad \zeta = \frac{\sqrt{2}}{\eta\sqrt{\lambda}}$$

$$masses: m_\Phi = \eta\sqrt{\lambda}, \quad m_A = e\eta$$

$$widths: \delta_\Phi \sim \frac{1}{m_\Phi}, \quad \delta_A \sim \frac{1}{m_A}$$

$$Bogomol'nyi\ parameter: \ \alpha_b \equiv \frac{m_A^2}{m_\Phi^2} = \frac{e^2}{\lambda}$$

$$Ginsburg-Landau\ parameter: \ \kappa = \frac{\nu}{\zeta} = \frac{\sqrt{\lambda}}{\sqrt{2}e}$$

The two parameters α_b and η play an important role by the calculation of the forces between the vortices. For small r, E can be approximated by $E \approx n^2 \frac{\ln\kappa}{e^2\hbar^2\nu^2}$ The field equations become

$$\partial_{rr}P = \frac{\partial_r P}{r} + e^2\eta^2 P X^2, \qquad \partial_{rr}X = -\frac{\partial_r X}{r} + n^2\frac{XP^2}{r^2} + \frac{1}{2}\lambda\eta^2 X(X^2-1) \tag{10}$$

In figure 3 a typical solution of the NO vortex string is visualized.

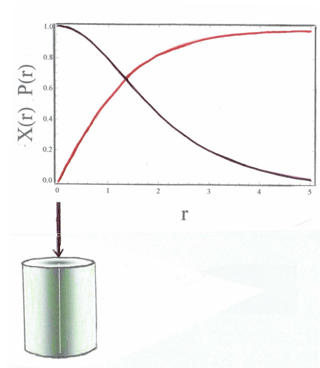

FIG. 3. Typical numerical solution of the Higgs (P) and gauge field (X).

A particular feature of these particle-like solutions is their topological structure characterized by an integer n, the topological charge, or winding number of the field. The topological charge can also be identified as the net number of the new type of particle. As can be seen from Eq. (9), the energy increases with n^2. For $n = 1$ we have the minimal energy situation, which is stable as it cannot decay into a topological trivial field. These field configurations are also called solitons.

B. The approximation

In order to study perturbations in the model, one can apply the multiple-scale approximation, very suited at high curvature situations[18–20]. The method is called a "two-timing" method, because one considers the relevant fields V_i in point $\mathbf{x}$ on a manifold M dependent on different scales $(\mathbf{x}, \xi, \chi, ...)$:

$$V_i = \sum_{n=0}^{\infty} \frac{1}{\omega^n} F_i^{(n)}(\mathbf{x}, \xi, \chi, ...). \tag{11}$$

Here ω represents a dimensionless parameter, which will be large (the "frequency", $\omega >> 1$). So $\frac{1}{\omega}$ is a small expansion parameter. Further, $\xi = \omega\Theta(\mathbf{x})$, $\chi = \omega\Pi(\mathbf{x})$, ... and $\Theta, \Pi, ...$ scalar (phase) functions on M. The parameter $\frac{1}{\omega}$ can be the ratio of the characteristic wavelength of the perturbation to the characteristic dimension of the background. On warped spacetimes it could also be the ratio of the extra dimension y to the background dimension or even both. When one substitutes the expansions of the field variables

$$\begin{aligned} g_{\mu\nu} &= \bar{g}_{\mu\nu}(\mathbf{x}) + \frac{1}{\omega}h_{\mu\nu}(\mathbf{x}, \xi) + \frac{1}{\omega^2}k_{\mu\nu}(\mathbf{x}, \xi) + ..., \\ A_\mu &= \bar{A}_\mu(\mathbf{x}) + \frac{1}{\omega}B_\mu(\mathbf{x}, \xi) + \frac{1}{\omega^2}C_\mu(\mathbf{x}, \xi) + ..., \\ \Phi &= \bar{\Phi}(\mathbf{x}) + \frac{1}{\omega}\Psi(\mathbf{x}, \xi) + \frac{1}{\omega^2}\Xi(\mathbf{x}, \xi) + ..., \end{aligned} \tag{12}$$

into the equations, one obtains first order equations in $u = t - r$ for the first and second order perturbations[22–24]. They are of the form

$$\partial_u \dot{\mathbf{U}}_1 = \bar{\mathbf{A}}, \qquad \partial_u \dot{\mathbf{U}}_2 = \bar{D}_1 \dot{\mathbf{U}}_2 + D_2 \dot{\mathbf{U}}_1 + D_3 \tag{13}$$

where $\bar{A}$ and $\bar{D}_1$ depends solely on the background fields, while D_2, D_3 depend on the first order perturbations and background fields. In principle one could push the approximation to higher orders. In this way one obtains a wavelike approximation which is asymptotically finite[18]. To highest order in ω, the equations deliver constraints on h, B and

C. In other approximations, they are a priori used as gauge conditions. Moreover, the original symmetry on the gauge field will be broken by the appearance of B_t besides B_φ.

In our approximation scheme we can gain a lot of insight in the behavior of the clustering of vortices when gravity is present. The equations (Eq. (13)) are hard to solve. However, the energy-momentum tensor components can tell us a lot about the behavior of the model.

C. Excitation of the vortices and the quasar link

In the expansion of Eq. (12) we parameterized the scalar field in subsequent orders as

$$\bar{\Phi} = \eta\bar{X}(t,r)e^{in_1\varphi}, \qquad \Psi = Y(t,r,\xi)e^{in_2\varphi}, \qquad \Xi = Z(t,r,\xi)e^{in_3\varphi}. \tag{14}$$

It turns out that the solution to second order is no longer axially symmetric. There appear terms like $\sin(n_2 - n_1)\varphi$ in the field equations. The most interesting information can be found in the energy-momentum tensor components

$$^4T^{(0)}_{t\varphi} = \bar{X}\bar{P}\dot{Y}n_1\mathbf{sin}[(n_2 - n_1)\varphi], \tag{15}$$

$$^4T^{(0)}_{tt} = \dot{Y}^2 + \dot{Y}(\partial_t\bar{X} + \partial_r\bar{X})\mathbf{cos}[(n_2 - n_1)\varphi] + \frac{e^{2\bar{\psi}}}{\bar{W}_1^2 r^2\epsilon}\Big(\epsilon\dot{B}^2 + n_1\dot{B}(\partial_r\bar{P} + \partial_t\bar{P})\Big), \tag{16}$$

While $^4\bar{T}_{t\varphi} = 0$, we conclude from Eq. (16) that the axial symmetry is broken already to first order. The energy

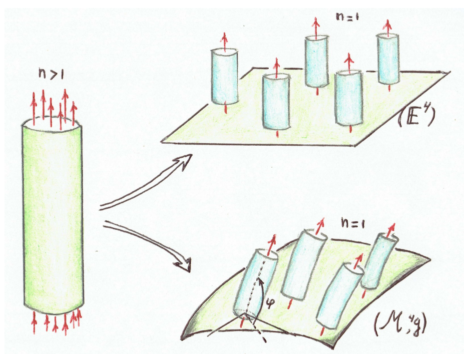

FIG. 4. Excitation and decay of a high multiplicity vortex into correlated vortices of unit flux $n = 1$. Top: the Abrikosov lattice in Euclidean space. Bottom: correlated vortices with preferred azimuthal angle φ in curved spacetime after the symmetry breaking.

$^4T^{(0)}_{tt}$ contribution to first order contains the warp factor in the denominator. So the energy depends crucially on the age of the universe. Terms in the energy can dominate at early times and are negligible at late times in the evolution of the universe. In the second order contributions there appear terms like $\cos(n_3 - n_1)\varphi$[24]. The azimuthal-angle dependency are expressed in trigonometrical functions with extrema which differ $mod(\frac{\pi}{k})$. After the excitation of the vortex with multiplicity n, it will decay into n vortices of unit flux in a regular lattice (figure 4). In flat spacetime, without gravity (upper picture in figure 4), this arrangement is experimentally observed. The Abrikosov vortices form a hexagonal lattice such that the energy is minimal. This process depends on the Bogomol'nyi parameter. In the special case of $\alpha_b = 1$ (mass of scalar and gauge field are equal) are the forces between the vortices easier to understand. It was a great achievement of Bogomol'nyi[25] to find the decoupled equations

$$\partial_{rr}X = -\frac{1}{r}\partial_r X + \frac{1}{X}\partial_r X^2 + \frac{1}{2}e^2\eta^2 X(X^2 - 1), \qquad P = \frac{r}{n\eta X}\partial_r X \tag{17}$$

Without the Bogomol'nyi equations it is difficult to understand the cancellation of the forces. The movement of the gauged vortices are even harder to understand[6].

There is another characterization of the winding number. It is the total vortex number, i.e., the number of points in the plane with multiplicity taken into count where $\Phi = 0$. The zero's of Φ are then a set of n isolated points $z_i, i = 0..n$ in $\mathbb{C}$ such that $\Phi(z, z^*) \sim c_j(z - z_j)^{n_j}$ with n_j the multiplicity of z_j and $n = \sum_{z_j} n_j$. This n-vortex solution represents a finite energy configuration with n flux quanta, provided Φ and A satisfy the boundary conditions

$$\lim_{r\to 0} X = 0, \quad \lim_{r\to 0} P = 1, \quad \lim_{r\to\infty} X = 1, \quad \lim_{r\to\infty} P = 0 \tag{18}$$

The collection of n vortices of unit flux is energetically more appealing than a n-flux vortex. The energy density is peaked around the zero's of Φ. Hence they can be identified by the location of the vortices. In general, one must solve the time-dependent GL equations in order to get insight in the stability issues. In this case there is a gradient flow, which makes the analysis very complicated. The PDE's are badly nonlinear and one relies often on numerical simulation. The temporarily broken axial symmetry will be the onset of emission of electro-magnetic and gravitational waves. From Eq. (15) we see that the angular momentum will fade away when n_2 approaches n_1 and the axial symmetry is restored. The first and second order perturbations of the scalar and gauge fields in higher winding number decay into NO strings of n=1. In order to understand the azimuthal-angle (φ) preference, one must consider the terms of ${}^4T_{\varphi\varphi}$, for example

$$ {}^4T^{(0)}_{\varphi\varphi} = e^{-2\gamma} r^2 \dot{Y}(\partial_t \bar{X} - \partial_r \bar{X}) \mathbf{cos}[(n_2 - n_1)\varphi] + \frac{n_1 e^{2\bar{\psi}-2\bar{\gamma}}}{\bar{W}_1^2 \epsilon} \dot{B}(\partial_r \bar{P} - \partial_t \bar{P}), \tag{19} $$

$$ \begin{aligned} {}^4T^{(1)}_{\varphi\varphi} = {} & e^{-2\gamma} r^2 \dot{Z}(\partial_t \bar{X} - \partial_r \bar{X}) \mathbf{cos}[(n_3 - n_1)\varphi] + \frac{e^{2\bar{\psi}-2\bar{\gamma}}}{\bar{W}_1^2 \epsilon} n_1 \dot{C}(\partial_r \bar{P} - \partial_t \bar{P}) + e^{-2\bar{\gamma}} r^2 \dot{Y}(\partial_t Y - \partial_r Y) + \bar{X}^2 n_1 \bar{P} \epsilon B \\ & + \Big[\frac{e^{2\bar{\psi}-2\bar{\gamma}}}{\bar{W}_1^2} \dot{Y}(\partial_t \bar{X} - \partial_r \bar{X})(h_{44} + e^{-2\bar{\gamma}} r^2 h_{11}) + n_1 \bar{X} \bar{P} Y (n_2 - n_1 + n_1 \bar{P}) + \frac{1}{2}\beta e^{-2\bar{\psi}} \bar{W}_1^2 r^2 \bar{X} Y (\eta^2 - \bar{X}^2) \\ & + e^{-2\bar{\gamma}} r^2 (\partial_t \bar{X} \partial_t Y - \partial_r \bar{X} \partial_r Y)\Big] \mathbf{cos}[(n_2 - n_1)\varphi] + \frac{e^{4\bar{\psi}-4\bar{\gamma}}}{\bar{W}_1^4 r^2 \epsilon^2} \Big[r^2 \epsilon \dot{B} n_1 (\partial_r \bar{P} - \partial_t \bar{P}) + \frac{1}{2} r^2 n_1^2 (\partial_r \bar{P}^2 - \partial_t \bar{P}^2) \\ & + \frac{1}{2} \bar{W}_1^2 \epsilon^2 e^{2\bar{\psi}} (\partial_t \bar{X}^2 - \partial_r \bar{X}^2)\Big] h_{11} + \Big[\frac{1}{2\bar{W}_1^2} e^{2\bar{\psi}-2\bar{\gamma}} (\partial_t \bar{X}^2 - \partial_r \bar{X}^2) - \frac{1}{8}\beta(\bar{X}^2 - \eta^2)^2\Big] h_{44}. \end{aligned} \tag{20} $$

$T_{\varphi\varphi}$ plays an important role in the interaction of the strings. Positive terms in the expression indicate "pressure" in the direction of the Killing vector field $(\frac{\partial}{\partial\varphi})^i$ (and negative "tension"). The result is that the interaction contribution can change sign dynamically (dependent of the warp factor). This Killing vector must be normalized such that, along a closed integral curve, the parameter φ varies van 0 to 2π with $\varphi = 0$ and $\varphi = 2\pi$ identified. This will provide boundary conditions for the metric fields close to the axis of the string, such as $\partial_r(r^2 e^{-2\psi})(0) = 1$. We observe in the expressions of ${}^4T^{(0)}_{\varphi\varphi}$ and ${}^4T^{(0)}_{t\varphi}$ that when $sin(n_2 - n_1)\varphi$ becomes zero, $cos(n_2 - n_1)\varphi$ has its maximum. So there is an emergent imprint of a preferred azimuthal angle φ on the lattice of vortices when the ground state is reached (n=1). This effect can also be seen in the ${}^4T^{(0)}_{zz}$ component which is not equal to $-{}^4T^{(0)}_{tt}$ as is the case in static models. The second order contribution ${}^4T^{(1)}_{\varphi\varphi}$ contains terms like $cos(n_3 - n_1)\varphi$ and produces a complicated extrema[24].

The recently observed alignment of the spinning axes of quasars in LQG's on Mpc-scales can be explained by our model. The observations were carried out at the European Southern Observatory, Paranal with the Very Large Telescope equipped with the FORS2 instrument. There was a confirmation of the alignment for radio galaxies by the Giant Metrewave Radio Telescope in the ELAIS-N1 field. This curious effect cannot be the result of statistical fluctuations[26]. The origin must be found in the early universe just after the symmetry breaking, as described in our model. Specially, the two preferred orientations perpendicular to each other in quasar groups of less richness could be the second order effect in our model by the appearance of the trigonometrical terms with periodicity difference of $\frac{\pi}{2}$. The correlated $n = 1$ vortices with preferred azimuthal angle, emerged on a correlation length smaller than the horizon on that moment and took place at the Ginzburg temperature $\sim \frac{1}{\beta\eta}$. These correlated regions will survive to later times, because at this moment the gravity contribution from the 5D bulk comes into play. The warp factor (see Figure 2) will have different contributions to the field equations for different times. The mass per unit length will contain the warp factor. Just after the symmetry breaking, the vortex will acquire a huge mass $G\mu > 1$ and will initiate the perturbations of high-frequency and justifies our high-frequency approximation. This is the reason that the regions with $(n = 1, \varphi = \varphi_0)$ will stick together and are observed in LQG's with aligned polarization axes[9, 10]. Some specific features of this alignment which could be explained with our model, must be confirmed by more observations on quasars and radio sources at high redshift. Alignment at high redshifts would confirm that the mechanism took place indeed in the early universe.

D. Breaking of the axial symmetry from a different viewpoint

Self-gravitating objects in equilibrium exhibit a striking analogue with the mathematical model of the Maclaurin-Jacobi sequences and its bifurcation points[27]. Bifurcation points that are of particular interest to us here are those marking the onset secular instability, i.e., the dynamical breaking of axially symmetry (or better formulated: the spacetime possesses 2 in stead of 3 Killing vectors). This means the appearance of an off-diagonal metric function.

In our model it is the transition from the Weyl metric (after the substitution $t \to iz, z \to it$) to the Papapetrou metric, expressed by the appearance of the $T_{t\varphi}$ components in first and second order. It is remarkable that this symmetry breaking can be compared with the second order phase transition in type II superconductivity[28–30], which is the basis of our model (see also Slagter[24] for more details). An initial axially symmetric configuration, as is the case in our perturbative model, can dynamically spontaneously be broken, where equatorial eccentricity plays the role of order-parameter. The equatorial eccentricity $\varepsilon \equiv \frac{b}{a}$, with b and a the two equatorial axes, can be expressed through the azimuthal-angle $\varphi(t)$. The particular orientation of the ellipsoid in the frame (r, φ, z) expressed through $\varphi_0 \equiv \varphi(t_0)$, will be at $t > t_0$ determined by the transformation $\varphi \to \varphi_0 - Jt$, where J is the rotation frequency (circulation or "angular momentum") of the coordinate system. The angle φ_0 is fixed arbitrarily at the onset of symmetry breaking. This arbitrariness of φ_0, i.e., the orientation of the ellipsoid at $t = t_0$ can be compared with the massless Goldstone-boson modes of the spontaneously broken symmetry of continuous groups. The phase transition take place on the same time scale that the vorticity is destroyed by dissipative mechanism and $\mathcal{J}$ is lost. The end point is a lower energy state that belongs to the Jacobi or Dedekind sequence of equilibrium ellipsoids[31]. In the original paper of Chandrasekhar and Lebovitz[28], in the Newtonian case, the deformations of the axisymmetric configuration by an infinitesimal nonaxisymmetric deformation is described in terms of a Lagrangian displacement $\varsigma^a(r, z, \varphi) = \bar{\varsigma}^a(r, z)e^{in\varphi}$, with n an integer. However, the real part of the $e^{in\varphi}$ must be put in by hand, in contrast to our result: it appears in a perturbative way as a first and second order effect. The temporarily broken axial symmetry will be the onset of emission of electro-magnetic and gravitational waves, while the string relaxes to the NO configuration. It is a consequence of the coupled system of PDE's that a high-frequency scalar field can create through an electro-magnetic field, a high frequency gravitational field and conversely. It is the appearance of the term $sin(n_2 - n_1)\varphi$ in the first order term ${}^4T^{(0)}_{t\varphi}$ (Eq.(15)) and explained in section 2c, which triggers this angular momentum and the axially symmetry will be restored when n_2 becomes equal to n_1 again. The second order contribution ${}^4T^{(1)}_{tt}$ shows terms like $\dot{B}^2 h_{44}$[24], indicating the interaction between the high-frequency EM and gravitational waves. It contains the warp factor in the denominator. In the early stages of the universe W_1 is still small and the term is significant. As time increases, it will fade away.

III. RELATION WITH CONFORMAL INVARIANCE

In the preceding sections we found that the warp factor $\mathcal{W}$ plays the role of a "scaling" factor, different at different epochs in time. There is, however, another interpretation, related to the dilaton field in conformal invariant gravity theory. The brane-part $W_1(t, r)$ could be solved from the 5D Einstein equations and was given in Eq.(6). The differential equation could be separated and reads (we rename, for historical reason, W_1 in ω, not to confuse with the expansion parameter in Eq.(11))

$$\partial_{tt}\omega = \partial_{rr}\omega + \frac{1}{\omega}\Big((\partial_r\omega)^2 - (\partial_t\omega)^2\Big) + \frac{2}{r}\partial_r\omega. \tag{21}$$

One can then write the spacetime[32]

$$g_{\mu\nu} = \omega^2 W_2^2 \tilde{g}_{\mu\nu} + n_\mu n_\nu \tag{22}$$

where the dilaton is conformally coupled to gravity and embedded in a smooth $M_4 \otimes R$ manifold by the action

$$\mathcal{I} = \int d^4x\sqrt{-\tilde{g}}\Big\{-\frac{1}{12}\Big(\tilde{\Phi}\tilde{\Phi}^* + \bar{\omega}^2\Big)\tilde{R} - \frac{1}{2}\Big(\mathcal{D}_\alpha\tilde{\Phi}(\mathcal{D}^\alpha\tilde{\Phi})^* + \partial_\alpha\bar{\omega}\partial^\alpha\bar{\omega}\Big) - \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} - V(\tilde{\Phi}, \omega) - \frac{1}{36}\kappa_4^2\Lambda_4\bar{\omega}^4\Big\} \tag{23}$$

We wrote $\Phi = \frac{1}{\omega}\tilde{\Phi}$ and Newton's constant is absorbed in a redefinition of $\bar{\omega}$. ω is taken complex in order to make the dilaton field comparable with the scalar field (see solution C in figure 1). A term $\sim \omega^4$ can be added to the action. Such a term could play a role in the generation of a cosmological constant. This action is local (Weyl-) conformal invariant by the transformation

$$\tilde{g}_{\mu\nu} \to \Omega^2\tilde{g}_{\mu\nu}, \qquad \bar{\omega} \to \frac{1}{\Omega}\bar{\omega}, \qquad \tilde{\Phi} \to \frac{1}{\Omega}\tilde{\Phi}. \tag{24}$$

when there is no mass term in V, because a mass term spoils the tracelessness of the energy momentum tensor. One could say that the conformal symmetry is spontaneously broken, just as the gauge symmetry of in the Brout-Englert-Higgs mechanism is spontaneously broken.

However, there will be no singular behavior when the former "scale"-function ω approaches zero (the small distance limit), because the Einstein field equations will contain in the dominator the term $\omega^2 + |\Phi|^2$ (the scalar field and dilaton field are treated here on equal footing). So we could have a regular description of gravity at very small distances[33–36].

The action appears to be entirely renormalizable for the dilaton field. After integrating over ω but not yet over $\tilde{g}_{\mu\nu}$, the resulting action stays local conformal invariant. The problem is that $\tilde{g}_{\mu\nu}$ in Eq.22 is not flat. One could consider an addition gauge transformation on $\tilde{g}_{\mu\nu}$ to make it Ricci-flat. Calculations on renormalizability and the appearance of anomalies would then be simplified[34].

On a flat background the choice of ω is unique. In curved spacetime it is fixed only if we know the evolution of spacetime and after choosing a coordinate frame. We conjecture that in our warped 5D model is it fixed by the evolution of the Einstein equations: the dilaton field plays the role of the warp factor.

It is a tantalizing idea to connect the mass generation in the Higgs-mechanism with the tracelessness of $T_{\mu\nu}$ (see for example the treatment of Mannheim[36]) and the cosmological constant problem. If one omits a kinematic mass term ($\sim \beta\eta^2 X^2\omega^2$) in the action (so $T_{\mu\nu}$ is traceless) and include a fermion field in the action, then one can dynamically generate massive particles without breaking the tracelessness of $T_{\mu\nu}$. Moreover, the cosmological constant can naturally arise in these dynamical mass theories and its value is constrained.

There is another possible way out for the breaking of the tracelessness of $T_{\mu\nu}$. In our 5D warped spacetime, the trace of the energy momentum tensor[32]

$$\frac{1}{\bar{\omega}^2 + X^2}\Big[16\kappa_4^2\beta\eta^2 X^2\bar{\omega}^2 - \kappa_5^4 n^4\Big(\frac{(\partial_r P)^2 - (\partial_t P)^2}{r^2\epsilon^2}\Big)^2 e^{8\tilde{\psi}-4\tilde{\gamma}}\Big] \tag{25}$$

will contain contributions from $\mathcal{S}$ (see Eq.(3)). The demand of tracelessness will deliver constraints on the parameters of the model in a dynamical way. Newton's constant, for example, reappears by the conformal breaking (note that this constant is hidden in the effective quartic interaction term for the Φ field). It is conjectured that constraints on the small distance behavior, all the physical constants, including the masses and cosmological constant, are constrained to values that are computable in terms of the Planck unit. For example , all β-coefficients of the renormalization group must vanish by the adjustment of the coupling constants (note that the zeros of the beta functions are isolated stationary points in quantum field theory).

In the conformal model there are still many problems unsolved, for example, the anomalies, which must be constrained to cancel out. Further, the black hole complementarity in conformal gravity is not yet well-understood[33].

IV. CONCLUSION

By considering an axially symmetric warped five-dimensional warped spacetime, were the standard model fields are confined to the brane, we find in a nonlinear approximation, an emergent azimuthal-angle dependency of Nielsen-Olesen vortices just after the symmetry breaking at GUT scale. Using a approximation scheme, the azimuthal-angle dependency appears in the first and second order field equations as trigonometrical functions $\sin(n_i - n_j)\varphi$ and $\sin(n_i - n_j)\varphi(i > j)$, with n_i the multiplicities of subsequent perturbation terms of the scalar field. Vortices with high multiplicity decay into a lattice with entangled Abrikosov vortices. The stability of this lattice of correlated flux $n = 1$ vortices with preferred azimuthal-angle is guaranteed by the contribution from the bulk spacetime by means of the warp factor: the cosmic string becomes super-massive for some time during the evolution. We used this azimuthal-angle correlation for the explanation of the recently observed alignment of polarization axes of quasars in large quasar groups. The detailed behavior of this alignment can be explained with our model. The two different orientations perpendicular to each other in quasars groups of less richness could be a second order effect in our model.

When gravity is coupled to standard model fields and one demands the validity on all distance scales, one runs into problems. These are: the dark energy problem, the cosmological constant problem, the hierarchy problem and the problem how probe the small distance structure of our spacetime. Conformal invariance could be the solution for at least some of these problems. It could be the missing symmetry of nature. In our model, by identifying the warp factor as a dilaton field, one will not encounter singular behavior when the dilaton field becomes very small. At present time, the warp-like manifestation of the dilaton field describes the exponential expansion of our universe. Moreover, the exceptional smallness of the cosmological constant, $\Lambda \asymp\sim 10^{-120}$ compared to the calculated vacuum energy could be explained in the warped 5D spacetime by the warp factor.

More data of high-redshift quasars will be needed in order to test the second order effect predicted in our model.

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