Inflation Driven By Scalar Field And Solid Matter

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Solid inflation is a cosmological model where inflation is driven by fields which enter the Lagrangian in the same way as body coordinates of a solid matter enter the equation of state, spontaneously breaking spatial translational and rotational symmetry. We construct a simple generalization of this model by adding a scalar field with standard kinetic term to the action. In our model the scalar power spectrum and the tensor-to-scalar ratio do not differ from the ones predicted by the solid inflation qualitatively, if the scalar field does not dominate the solid matter. The same applies also to the size of the scalar bispectrum measured by the non-linearity parameter, although our model allows it to have different shapes. The tensor bispectra predicted by the two models do not differ from each other in the leading order of the slow-roll approximation. In the case when contribution of the solid matter to the stress-energy tensor is much smaller than the contribution from the scalar field, the tensor-to-scalar ratio and the non-linearity parameter are amplified by factors ϵ^{-1} and ϵ^{-2} respectively. The full version of the paper can be found in [1].

Keywords: inflation; solid matter; quantum field theory in curved space-time.

1. Model Under Consideration

Multi-field models of inflation lead to "local" non-Gaussianity peaking at "squeezed" configuration of momenta ($k_1 \approx k_2 \gg k_3$). One of less standard examples of such models is *solid inflation*[2,3], driven by a three-component scalar field ϕ^I which enters the Lagrangian in the same way as body coordinates of solid matter enter the equation of state. Thus, the matter action is supposed to be invariant under internal translations and rotations,

$$\phi^I \to M^I{}_J \phi^J, \quad M^I{}_J \in SO(3), \quad \phi^I \to \phi^I + C^I, \quad C^I \in \mathbb{R}^3, \quad I, J = 1, 2, 3, \tag{1}$$

The simplest possible background configuration, $\phi^I = \delta^I_i x^i$, with x^i denoting spatial coordinates, breaks the spatial translational and rotational symmetry, but in a flat Friedmann–Robertson–Walker–Lematre universe it is invariant under the combined spatial-internal transformations. As shown by Endlich et al.[3], in this model there appears anisotropic dependence of the scalar bispectrum on how the squeezed limit is approached. Further development of the theory includes papers[4–7].

In our paper we study a combined inflationary model including scalar field φ with standard kinetic term and three-component scalar field ϕ^I with symmetries given above. The matter Lagrangian of the theory is

$$\mathcal{L} = -\frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi + F(\varphi, X, Y, Z), \quad X = \mathrm{Tr}B, \quad Y = \frac{\mathrm{Tr}(B^2)}{X^2}, \quad Z = \frac{\mathrm{Tr}(B^3)}{X^3}. \tag{2}$$

Following[3], we have introduced the three independent quantities X, Y and Z invariant under transformations (1) composed of the body metric $B^{IJ} = -g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J$.

(We have changed the sign of B^{IJ} in order to reconcile it with the signature of the metric tensor $(+\,-\,--)$, which we use.) Our model represents a straightforward combination of the solid inflation and basic single-field models. It can be considered as, for instance, a simple toy model of interaction of fields driving the solid inflation with fields of an effective field theory of the standard model.

In the inflationary model (2) the slow-roll parameter $\epsilon = -\dot{H}/H^2$ is

$$\epsilon = p + q - \frac{1}{3}pq, \quad p = \frac{\dot{\varphi}^2}{2M_{\rm Pl}^2 H^2}, \quad q = X\frac{F_X}{F}, \tag{3}$$

where p and q are the slow-roll parameters of the single-field inflation and the solid inflation respectively. In our work we restricted ourselves to the special case when both p and q are small.

2. Power Spectrum

From[8] we adopt the definition of the scalar quantity ζ that parametrizes the curvature perturbations. The corresponding power spectrum $\mathcal{P}_\zeta(k)$ is defined by the two-point function in the late-time limit, $\langle 0|\,\zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\,|0\rangle = [\mathcal{P}_\zeta(k_1)/(2k_1^3)](2\pi)^5\delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2)$. It is usually approximated by a power-law function, $\mathcal{P}_\zeta(k) \propto k^{n_S-1}$, where n_S, called the scalar spectral index, is close to one for a nearly flat spectrum. In the leading order of the slow-roll approximation the spectral tilt is

$$n_S - 1 = -2\frac{c_{L,e}^5\sigma p_e \epsilon_c^{(\delta\varphi)} + (\epsilon_e - p_e)\, p_c^{(\mathcal{U})}}{\epsilon_e + \left(c_{L,e}^5\sigma - 1\right) p_e}, \tag{4}$$

where

$$c_L = \sqrt{1 + \frac{2}{3}\frac{XF_{XX}}{F_X} + \frac{8}{9}\frac{F_Y + F_Z}{XF_X}} \tag{5}$$

is the longitudinal sound speed of medium filling the universe, $\sigma = e^{2N_{\min}\left(\epsilon_c^{(\delta\varphi)} - p_c^{(\mathcal{U})}\right)}$, $p^{(\mathcal{U})} = p - c_L^2 Q + \frac{1}{2}\eta_Q + \frac{5}{2}\eta_L$, $\epsilon^{(\delta\varphi)} = \epsilon + 2p + \frac{1}{3}\frac{F_{\varphi\varphi}}{H^2}$, $Q = \epsilon - p$, and both $\eta_Q = \dot{Q}/(\epsilon H)$ and $\eta_L = \dot{c}_L/(c_L H)$ have been assumed to be of the first order in the slow-roll parameters. The subscript e stands for quantities evaluated at the time τ_e when the inflation ends, $\tau_e \approx 0^-$, the subscript c stands for quantities evaluated at the reference time τ_c when the longest mode of observational relevance today with the wavenumber $k_{\min} \sim H_{\rm today}$ ($a_{\rm today} \equiv 1$) exits the horizon, and $N_{\min} \sim 60$ is the minimal number of e-foldings.

We have computed also the tensor spectral tilt and tensor-to-scalar ratio in our model. The results are

$$n_T - 1 = 2c_{L,c}^2\epsilon_c - 2(1 + c_{L,c}^2)p_c, \qquad r = \frac{\mathcal{P}_\gamma}{\mathcal{P}_\zeta} = \frac{4c_L^5\epsilon^2}{\epsilon + (c_L^5 - 1)\,p}. \tag{6}$$

3. Bispectrum

The three-point function of the scalar ζ can be computed with the use of the in-in formalism[9]. The scalar bispectrum $B_\zeta(k_1,k_2,k_3)$, defined by the relation $\langle\zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_\zeta(k_1,k_2,k_3)$, in our model consists of two parts,

$$B_\zeta(k_1,k_2,k_3) = F_Y B_\zeta^Y(k_1,k_2,k_3) + \mathcal{N}_\zeta c_{L,c}^2 \tilde{F}\tilde{B}_\zeta(k_1,k_2,k_3), \tag{7}$$

parametrized by three independent parameters of the theory, namely F_Y,

$$\tilde{F} = X\left(F_{XY}+F_{XZ}\right) = \pm\frac{M_{\rm Pl}}{\sqrt{2}}\sqrt{p}\left(F_{Y\varphi}+F_{Z\varphi}\right), \tag{8}$$

and $\mathcal{N}_\zeta c_{L,c}^2$, where $\mathcal{N}_\zeta$ is a number of the order of the number of e-foldings. Following the conventions of[10], we introduce dimensionless variables $x = k_2/k_1$ and $y = k_3/k_1$ and describe the bispectrum by the function $x^2y^2B_\zeta(1,x,y)$ defined in the region $1-x \le y \le x$, $1/2 \le x \le 1$, $0 \le y \le 1$. Shapes of the functions $x^2y^2B_\zeta^Y(1,x,y)$ and $x^2y^2\tilde{B}_\zeta(1,x,y)$ are depicted in the first two panels of fig. 1. (All functions in the figure are normalized to have value 1 in the equilateral limit, $x = y = 1$.) Our model

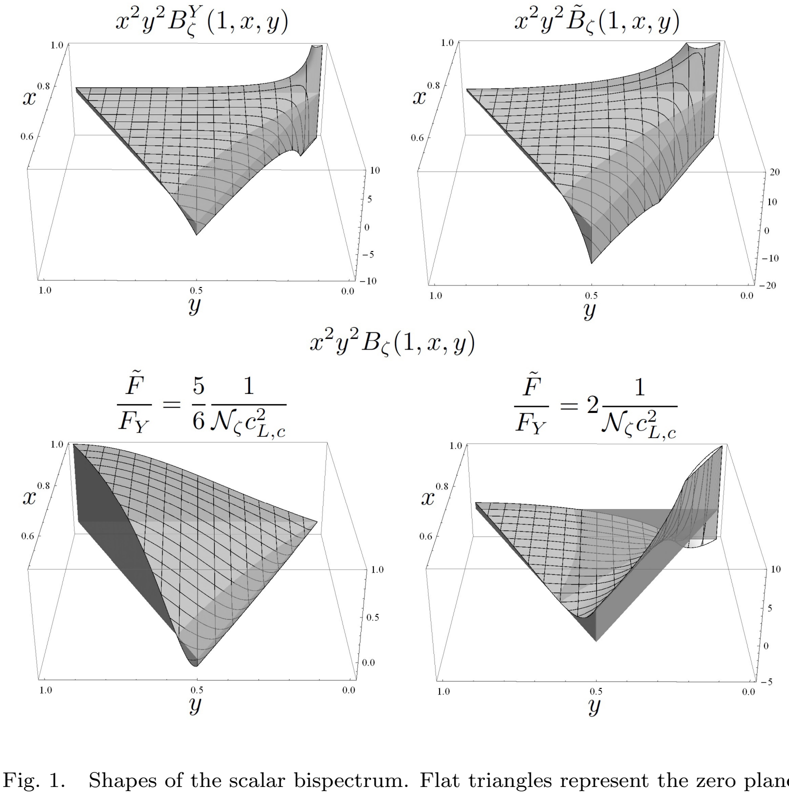

Fig. 1. Shapes of the scalar bispectrum. Flat triangles represent the zero plane.

with the additional degree of freedom allows for a wider range of different shapes of the bispectrum than the pure solid inflation. The overall bispectrum peaks in the squeezed limit, unless $\tilde{F}/F_Y = (5/6)\mathcal{N}_\zeta^{-1} c_{L,c}^{-2}$, when it peaks in the equilateral limit instead, see the third panel of fig. 1. When $\tilde{F}/F_Y$ exceeds $(5/6)\mathcal{N}_\zeta^{-1} c_{L,c}^{-2}$ the relative sign of the bispectrum in the squeezed limit and the bispectrum in the equilateral limit changes, as demonstrated in the fourth panel.

Following the definition (4) in[11], we find for the non-linearity parameter

$$f_{\rm NL} = \frac{\epsilon_c}{\left[\epsilon_c + \left(c_{L,c}^5 - 1\right) p_c\right]^2} \left(\frac{19415}{13122}\frac{1}{c_{L,c}^2}\frac{F_Y}{F} - \frac{5}{18}\mathcal{N}_\zeta \frac{\tilde{F}}{F}\right). \tag{9}$$

We can see that if $\epsilon - p \sim \epsilon \sim p$, the non-linearity parameter is of the order $f_{\rm NL} \sim (F_Y/F) c_L^{-2} \epsilon^{-1}$, the same as for the solid inflation without the scalar field, or $f_{\rm NL} \sim \mathcal{N}_\zeta (\tilde{F}/F)\epsilon^{-1}$. Supposing that $c_L^5 \sim \epsilon$ we have $f_{\rm NL} \sim (F_Y/F) c_L^{-2} \epsilon^{-3}$ or $f_{\rm NL} \sim \mathcal{N}_\zeta (\tilde{F}/F)\epsilon^{-3}$ if $\epsilon - p$ is of the order ϵ^2. The condition $\epsilon - p \lesssim \epsilon^2$ leading to an amplification of the non-linearity parameter can be written as $q \ll p$, which means that the contribution of solid matter to the overall stress-energy tensor is negligible in comparison to the contribution of the scalar field.

In our model the tensor bispectrum computed in the leading order of the slow-roll approximation does not differ from the tensor bispectrum in solid inflation. It is affected by the presence of the scalar field only in higher orders of the slow-roll approximation, which have not been included in our work.

Acknowledgments

I would like to thank Vladimír Balek for useful discussions and valuable comments. The work was supported by the grants VEGA 1/0985/16 and UK/36/2017.

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