Cosmology with High Redshift Probes: The GRBs Hubble Diagram

Marek Demianski,[1, 2] Ester Piedipalumbo,[3, 4, *] Disha Sawant,[5] and Lorenzo Amati[6]

[1] *Institute of Theoretical Physics, Faculty of Physics, University of Warsaw, Pasteura 5, 02-093 Warsaw, Poland*
[2] *Department of Astronomy, Williams College, Williamstown, MA 01267, USA*
[3] *Dipartimento di Fisica, Università degli Studi di Napoli Federico II, Compl. Univ. Monte S. Angelo, 80126 Naples, Italy*
[4] *I.N.F.N., Sez. di Napoli, Compl. Univ. Monte S. Angelo, Edificio 6, via Cinthia, 80126 - Napoli, Italy.*
[5] *Dipartimento di Fisica e Scienze della Terra, Università degli Studi di Ferrara*
[6] *INAF-IASF, Sezione di Bologna, via Gobetti 101, 40129 Bologna, Italy*

So far large and different data sets revealed the accelerated expansion rate of the Universe, which is usually assumed to be driven by the so called dark energy, that, according to recent estimates, provides about 70% of the total amount of the matter- energy in the Universe. The nature of dark energy is yet unknown. Whatever future data discover, the simple plot of the Hubble diagram (HD) as a function of redshift will remain one of the primary tool for cosmological investigations, as the conversion between redshift and distance depends on the specific parameters of the underlying models. We show that different dark energy models can be tested, by using a high redshift GRBS HD, obtained calibrating the Ep,i- Eiso correlation in long GRBs. It turns out that the Cosmological Constant model is not favored by the present data.

I. INTRODUCTION

Starting at the end of the 1990s, observations of high-redshift supernovae of type Ia (SNIa) revealed the current accelerated expansion of the Universe ([see e.g. 1–6]), which is driven by the so called dark energy. The so far proposed models of dark energy range from a non-zero cosmological constant (see for instance [9]), to a potential energy of some not yet discovered scalar field (see for instance [10]), or effects connected with the inhomogeneous distribution of matter and averaging procedures (see for instance [11]). Recentlyit has been examined the idea that dark energy originates from the backreaction of quantum fluctuations originating in the primordial inflationary universe. In this paper we analyze the observational constraints of a model proposed by Glavan, Prokopec and Starobinsky (GPS model,[17]). In this model an ultra-light, non-minimally coupled scalar field, which is a spectator field during inflation, but, during quantum fluctuations, naturally grow large such that, by the end of inflation, it reach super-Planckian values, and is viable for DE. Very briefly we consider different competitive cosmological scenarios:

i) an EOS empirically parametrized ,

ii) an exponential scalar field model for dark energy ,

iii) GPS model ,

iii) an early time dark energy model .

In our high-redshift investigation, extended beyond the supernova type Ia (SNIa) Hubble diagram, we use the Union2 SNIa data set, the gamma-ray burst (GRB) Hubble diagram, constructed by calibrating the correlation between the peak photon energy, $E_{\rm p,i}$, and the isotropic equivalent radiated energy, $E_{\rm iso}$ [15]. Here we take into account possible redshift evolution effects in the coefficients of this correlation, assuming that they can be modeled through power low terms. We consider also a sample of 28 measurements of the Hubble parameter, compiled in [16], Gaussian priors on the distance from the baryon acoustic oscillations (BAO), and the Hubble constant h. Our statistical analysis is based on Monte Carlo Markov Chain (MCMC) simulations to simultaneously compute the full probability density functions (PDFs) of all the parameters of interest.

II. COMPETITIVE DARK ENERGY MODELS

In the Friedman-Lemaitre-Robertson- Walker paradigm, all possibilities can be characterized by the dark energy EOS, $w(z)$. A prior task of observational cosmology is to search for evidence for $w(z) \neq -1$. If we assume that the dark energy evolves, the importance of its equation of state is significant and it determines the Hubble function $H(z)$, and any derivation of it is needed to obtain the observable quantities. Actually it turns out that:

$$H(z,\theta) = H_0\sqrt{(1-\Omega_m)g(z,\theta)+\Omega_m(z+1)^3}$$

where $g(z,\theta) = \frac{\rho_{de}(z)}{\rho_{de}(0)} = \exp^{3\int_0^z \frac{w(x,\theta)+1}{x+1}\,dx}$, $w(z,\theta)$ is any dynamical form of the dark energy EOS, and $\theta = (\theta_1, \theta_1.., \theta_n)$ are the EOS parameters. In the Chevalier-Polarski Linder (CPL) model [12, 13], the dark energy EOS given by

$$w(z) = w_0 + w_1 z(1+z)^{-1}\,, \tag{1}$$

* ester@na.infn.itl

A. An exponential scalar field

In this section the possible physical realization of dark energy is a cosmic scalar field, φ, minimally coupled to the usual matter action. Here we take into account the specific class of exponential–type potential; in particular we consider an exponential potential for which general exact solutions of the Friedman equations are known[20, 21]. Assuming that φ is minimally coupled to gravity, the cosmological equations are written as

$$H^2 = \frac{8\pi G}{3}(\rho_M + \rho_\varphi)\,,$$

$$\dot{H} + H^2 = -\frac{4\pi G}{3}(\rho_M + \rho_\varphi + 3(p_M + p_\varphi))\,,$$

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0\,.$$

Here

$$\rho_\varphi \equiv \frac{1}{2}\dot{\varphi}^2 + V(\varphi)\,, \quad p_\varphi \equiv \frac{1}{2}\dot{\varphi}^2 - V(\varphi)\,, \tag{2}$$

and

$$w_\varphi \equiv \frac{\dot{\varphi}^2 - 2V(\varphi)}{\dot{\varphi}^2 + 2V(\varphi)}\,. \tag{3}$$

We consider the potential

$$V(\varphi) \propto \exp\left\{-\sqrt{\frac{3}{2}}\varphi\right\}\,, \tag{4}$$

for which the general exact solution exists [20] and [21].

B. Observational tests of the GPS dark energy

At the late stage of evolution, at $z \leq 10$, the Universe is filled in with dark matter, baryonic matter and dark energy. Dark matter is usually assumed to be cold and collisionless so both types of matter could be treated as pressureless dust with mass-energy density ϱ_m. Dark matter and baryonic mater does not interact with dark energy, so both matter components and dark energy could be treated as non interacting perfect fluids. The continuity equation for matter in the FLRW model has the simple form

$$\dot{\varrho}_m + 3H\varrho_m = 0\,, \tag{5}$$

corresponding equation for the GPS dark energy is

$$\dot{\varrho}_{DE} + 3H(1 - \frac{\omega_0 a^\alpha}{\beta + a^\alpha})\varrho_{DE} = 0\,, \tag{6}$$

where $H = \dfrac{\dot{a}}{a}$ and ω_0 , α , β are constants. Integrating both equations and using the standard relation $a(z) = \frac{1}{1+z}$, we get

$$\varrho_m(z) = \varrho_M(0)(1+z)^3\,, \tag{7}$$

$$\varrho_{DE} = \varrho_{DE}(0)(1+z)^{3(1-\omega_0)}\big(\frac{1+\beta(1+z)^\alpha}{1+\beta}\big)^{3\frac{\omega_0}{\alpha}}\,, \tag{8}$$

where $\varrho_m(0)$ is the present density of matter and $\varrho_{DE}(0)$ is the present density of dark energy. The Hubble expansion rate is

$$\begin{aligned}&3H^2(z) = \varrho_m(0)(1+z)^3 + \\ &\varrho_{DE}(0)(1+z)^{3(1-\omega_0)}\big(\frac{1+\beta(1+z)^\alpha}{1+\beta}\big)^{3\frac{\omega_0}{\alpha}}\Big)\,.\end{aligned} \tag{9}$$

C. Early Dark Energy

In this section we consider a model characterized by a non negligible amount of dark energy at early times: these models are connected to the existence of scaling or attractor-like solutions, and they naturally predict a non-vanishing dark energy fraction of the total energy at early stages, Ω_e, which should be substantially smaller than its present value. Following [22, 23] we use a parametrized representation of the dark energy density fraction, Ω_{DE}, which depends on the present matter fraction, Ω_m, the early dark energy density fraction, Ω_e , and the present dark energy equation of state w_0:

$$\begin{aligned}&\Omega_{DE}(z,\Omega_m,\Omega_e,w_0) = \\ &\frac{\Omega_e\left(-\left(1-(z+1)^{3w_0}\right)\right) - \Omega_m + 1}{\Omega_m(z+1)^{-3w_0} - \Omega_m + 1} \\ &+\Omega_e\left(1-(z+1)^{3w_0}\right)\,.\end{aligned}$$

It turns out that the Hubble function takes the form:

$$\begin{aligned}&H^2(z,\Omega_m,\Omega_e,w_0,\Omega_\gamma,N_{eff}) = \\ &\Omega_{DE}(z,\Omega_m,\Omega_e,w_0) + \\ &+(z+1)^3\Omega_m + \\ &(z+1)^4\Omega_\gamma\left(\frac{7}{8}\left(\frac{4}{11}\right)^{\frac{4}{3}}N_{eff} + 1\right)\,.\end{aligned} \tag{10}$$

Here $N_{eff} = 3$ for three standard model neutrinos that were thermalized in the early Universe and decoupled well before electron-positron annihilation.

III. OBSERVATIONAL DATA

In our approach we use measurements on SNIa and GRB Hubble diagram, distance data from the BAO, and a list of 28 $H(z)$ measurements, compiled in [16].

A. Supernovae Ia

SNIa observations gave the first strong indication of the recent accelerating expansion of the Universe. First results of the SNIa teams were published by [3] and [2]. Here we consider the recently updated Supernovae Cosmology Project Union 2.1 compilation [26], which is an update of the original Union compilation and contains 580 SNIa, spanning the redshift range ($0.015 \leq z \leq 1.4$). We compare the theoretically predicted distance modulus $\mu(z)$ with the observed one through a Bayesian approach, based on the definition of the distance modulus in different cosmological models:

$$\mu(z_j) = 5\log_{10}(D_L(z_j, \{\theta_i\})) + \mu_0\,, \tag{11}$$

where $D_L(z_j, \{\theta_i\})$ is the Hubble free luminosity distance, and θ_i indicates the set of parameters that appear in different dark energy equations of state considered in our analysis. The parameter μ_0 encodes the Hubble constant and the absolute magnitude M. Given the heterogeneous origin of the Union data set, we used an alternative version of the χ^2:

$$\tilde{\chi}^2_{\rm SN}(\{\theta_i\}) = c_1 - \frac{c_2^2}{c_3}\,, \tag{12}$$

where

$$c_1 = \sum_{j=1}^{\mathcal{N}_{SNIa}} \frac{(\mu(z_j;\mu_0=0,\{\theta_i)\} - \mu_{obs}(z_j))^2}{\sigma^2_{\mu,j}}\,,$$

$$c_2 = \sum_{j=1}^{\mathcal{N}_{SNIa}} \frac{(\mu(z_j;\mu_0=0,\{\theta_i)\} - \mu_{obs}(z_j))}{\sigma^2_{\mu,j}}\,,$$

$$c_3 = \sum_{j=1}^{\mathcal{N}_{SNIa}} \frac{1}{\sigma^2_{\mu,j}}\,.$$

It is worth noting that

$$\chi^2_{\rm SN}(\mu_0, \{\theta_i\}) = c_1 - 2c_2\mu_0 + c_3\mu_0^2\,, \tag{13}$$

which clearly becomes minimum for $\mu_0 = c_2/c_3$, so that $\tilde{\chi}^2_{\rm SN} \equiv \chi^2_{\rm SN}(\mu_0 = c_2/c_3, \{\theta_i\})$.

B. Gamma-ray burst Hubble diagram

Gamma-ray bursts are visible up to high redshifts thanks to the enormous energy that they release, and thus may be good candidates for our high-redshift cosmological investigation. We performed our analysis using a new updated GRB Hubble diagram data set obtained by calibrating a 3-d $E_{\rm p,i}$– $E_{\rm iso}$–z relation. Actually, even if recent studies concerning the reliability of the $E_{\rm p,i} - E_{\rm iso}$ relation confirmed the lack, up to now, of any statistically meaningful evidence for a z dependence of the correlation coefficients [14], we include in the calibration terms representing the z-evolution, which are assumed to be power-law functions: $g_{iso}(z) = (1+z)^{k_{iso}}$ and $g_p(z) = (1+z)^{k_p}$ (see for instance[14]), so that $E^{'}_{\rm iso} = \dfrac{E_{\rm iso}}{g_{iso}(z)}$ and $E^{'}_{\rm p,i} = \dfrac{E_{\rm p,i}}{g_p(z)}$ are the de-evolved quantities. Therefore we consider a 3D correlation:

$$\log\left[\frac{E_{\rm iso}}{1\ {\rm erg}}\right] = b + a\log\left[\frac{E_{\rm p,i}}{300\ {\rm keV}}\right] + + (k_{iso} - a\,k_p)\log{(1+z)}\ . \tag{14}$$

In order to calibrate our de-evolved relation we apply the same local regression technique previously adopted ([14, 15]), but we consider a 3D Reichart likelihood:

$$\begin{aligned} &L^{3D}_{Reichart}(a, k_{iso}, k_p, b, \sigma_{int}) = \\ &\frac{1}{2}\frac{\sum\log{(\sigma^2_{int} + \sigma^2_{y_i} + a^2\sigma^2_{x_i})}}{\log{(1+a^2)}} \\ &+ \frac{1}{2}\sum\frac{(y_i - ax_i - (k_{iso}-\alpha)z_i - b)^2}{\sigma^2_{int} + \sigma^2_{x_i} + a^2\sigma^2_{x_i}}\,, \end{aligned} \tag{15}$$

where $\alpha = a\,k_p$. We also used the MCMC method to maximize the likelihood and ran five parallel chains and the Gelman-Rubin convergence test. We found that $a = 1.87^{+0.08}_{-0.09}$, $k_{iso} = -0.04 \pm 0.1$; $\alpha = 0.02 \pm 0.2$; $\sigma_{int} = 0.35^{+0.02}_{-0.03}$, so that $b = 52.8^{+0.03}_{-0.06}$. After fitting the correlation and estimating its parameters, we used them to construct the GRB Hubble diagram.

C. Baryon acoustic oscillations data

Baryon acoustic oscillations data are promising standard rulers to investigate different cosmological scenarios and models. They are related to density fluctuations induced by acoustic waves that are created by primordial perturbations. To use BAOs as a cosmological tool, we define:

$$d_z = \frac{r_s(z_d)}{d_V(z)}\,, \tag{16}$$

where z_d is the drag redshift, $r_s(z)$ is the comoving sound horizon ,

$$r_s(z) = \frac{c}{\sqrt{3}}\int_0^{(1+z)^{-1}} \frac{da}{a^2H(a)\sqrt{1+(3/4)\Omega_b/\Omega_\gamma}}\,,$$

and $d_V(z)$ the volume distance. Moreover, BAO measurements in spectroscopic surveys allow to directly estimate the expansion rate H(z), converted into the quantity $D_H(z) = \dfrac{c}{H(z)}$, and put constraints on the comoving angular diameter distance $D_M(z)$.

D. H(z) measurements

The measurements of Hubble parameters are a complementary probe to constrain the cosmological parameters and investigate the dark energy [16]. The Hubble parameter depends on the differential age of the Universe as a function of redshift and can be measured using the so-called cosmic chronometers. dz is obtained from spectroscopic surveys with high accuracy, and the differential evolution of the age of the Universe dt in the redshift interval dz can be measured provided that optimal probes of the aging of the Universe, that is, the cosmic chronometers, are identified. The most reliable cosmic chronometers at present are old early-type galaxies that evolve passively on a timescale much longer than their age difference, which formed the vast majority of their stars rapidly and early and have not experienced subsequent major episodes of star formation or merging. Moreover, the Hubble parameter can also be obtained from the BAO measurements. We used a list of 28 $H(z)$ measurements, compiled in [16].

IV. STATISTICAL ANALYSIS

To test the cosmological parameters described above, we use a Bayesian approach based on MCMC method. In order to set the starting points for our chains, we first performed a preliminary and standard fitting procedure to maximize the likelihood function $\mathcal{L}(\mathbf{p})$:

$$\begin{aligned}\mathcal{L}(\mathbf{p}) \propto & \frac{\exp\left(-\chi^2_{SNIa/GRB}/2\right)}{(2\pi)^{\frac{\mathcal{N}_{SNIa/GRB}}{2}}|\mathbf{C}_{SNIa/GRB}|^{1/2}} \times \\ & \frac{\exp\left(-\chi^2_{BAO}/2\right)}{(2\pi)^{\mathcal{N}_{BAO}/2}|\mathbf{C}_{BAO}|^{1/2}} \times \\ & \times \frac{1}{\sqrt{2\pi\sigma^2_{\omega_m}}}\exp\left[-\frac{1}{2}\left(\frac{\omega_m-\omega_m^{obs}}{\sigma_{\omega_m}}\right)^2\right] \\ & \times \frac{1}{\sqrt{2\pi\sigma^2_{h}}}\exp\left[-\frac{1}{2}\left(\frac{h-h_{obs}}{\sigma_{h}}\right)^2\right] \\ & \frac{\exp\left(-\chi^2_{H}/2\right)}{(2\pi)^{\mathcal{N}_{H}/2}|\mathbf{C}_{H}|^{1/2}} \\ & \times \frac{1}{\sqrt{2\pi\sigma^2_{\mathcal{R}}}}\exp\left[-\frac{1}{2}\left(\frac{\mathcal{R}-\mathcal{R}_{obs}}{\sigma_{\mathcal{R}}}\right)^2\right] . \end{aligned} \tag{17}$$

Here

$$\chi^2(\mathbf{p}) = \sum_{i,j=1}^{N}\left(x_i - x_i^{th}(\mathbf{p})\right) C_{ij}^{-1}\left(x_j - x_j^{th}(\mathbf{p})\right) , \tag{18}$$

$\mathbf{p}$ is the set of parameters, N is the number of data points, x_i is the $i-th$ measurement; $x_i^{th}(\mathbf{p})$ indicate the theoretical predictions for these measurements and depend on the parameters $\mathbf{p}$. C_{ij} is the covariance matrix (specifically, $\mathbf{C}_{SNIa/GRB/H}$ indicates the SNIa/GRBs/H covariance matrix); $(h^{obs}, \sigma_h) = (0.742, 0.036)$ [Riess et al. 2009], and $(\omega_m^{obs}, \sigma_{\omega_m}) = (0.1356, 0.0034)$ [8]. It is worth noting that the effect of our prior on h is not critical at all so that we are certain that our results are not biased by this choice. The term $\frac{1}{\sqrt{2\pi\sigma^2_{\mathcal{R}}}}\exp\left[-\frac{1}{2}\left(\frac{\mathcal{R}-\mathcal{R}_{obs}}{\sigma_{\mathcal{R}}}\right)^2\right]$ in the likelihood (18) considers the shift parameter $\mathcal{R}$:

$$\mathcal{R} = H_0\sqrt{\Omega_M}\int_0^{z_\star}\frac{dz'}{H(z')} , \tag{19}$$

where $z_\star = 1090.10$ is the redshift of the surface of last scattering [29], [30] . According to the Planck data $(\mathcal{R}_{obs}, \sigma_{\mathcal{R}}) = (1.7407, 0.0094)$.

Finally, the term $\frac{\exp\left(-\chi^2_{H}/2\right)}{(2\pi)^{\mathcal{N}_{H}/2}|\mathbf{C}_{H}|^{1/2}}$ in Eq. (18) is the likelihood relative to the measurements of $H(z)$. For each cosmological model we sample its space of parameters, by running five parallel chains and use the Gelman - Rubin diagnostic approach to test the convergence.

V. DISCUSSION AND CONCLUSIONS

The $E_{\rm p,i} - E_{\rm iso}$ correlation has significant implications for the use of GRBs in cosmology. Here we explored a 3D Amati relation in a way independent of the cosmological model, and taking into account a possible redshift evolution effects of its correlation coefficients [14] parametrized as power low terms: $g_{iso}(z) = (1+z)^{k_{iso}}$ and $g_p(z) = (1+z)^{k_p}$. Low values of k_{iso} and k_p would indicate negligible evolutionary effects. Using the recently updated data set of 162 high-redshift GRBs, we applied a local regression technique to estimate the distance modulus using the recent Union SNIa sample (Union2.1). The derived calibration parameters are statistically fully consistent with the results of our previous work [14, 20], and confirm that the correlation shows, at this stage, only weak indication of evolution. The fitted calibration parameters have been used to construct a calibrated GRB Hubble diagram, which we adopted as a tool to constrain different cosmological models: we considered the CPL parameterization of the EOS, an exponential dark energy scalar field, and, finally a model with dark energy at early times. To compare these models we assumed that the CPL is true and checked the occurrence of $\chi^2_{EDE/Quintessence/GPS} < \chi^2_{CPL}$, varying the parameters specific of the EDE, exponential and GPS scalar field models respectively. Our statistical analysis indicates that the GPS models seems slightly favored with respect to the others. It means that to further restrict different models of dark energy it will be necessary to increase the precision of the Hubble diagram at high redshift, and to perform more detailed analysis of the influence of dark energy on the process of formation of large scale structure and in particular on its late evolution at $z < 2$. Future GRB missions, like, e.g., the proposed THESEUS observatory [31], will increase substantially the number of GRB usable to construct the$E_{\rm p,i} - E_{\rm iso}$ correlation up to redshift $z \simeq 10$ and will allow better cosmological investigations.

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