Constraint on the equation of state via supernova gravitational waves

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We focus on spacetime oscillations, the so-called w-modes, of gravitational waves emitted from a protoneutron star in the postbounce phase of core-collapse supernovae. By adopting numerical results from recent relativistic three-dimensional supernova models, we find that the w_1-mode frequency multiplied by the radius of the protoneutron star is expressed as a linear function with respect to the stellar compactness insensitively to the nuclear equation of state. Combining with another universal relation of the f-mode oscillations, it is shown that the time dependent mass-radius relation of the protoneutron star can be obtained by observing both the f- and w_1-mode gravitational waves simultaneously. That is, the simultaneous detection of the two modes could provide a new probe into finite-temperature nuclear equation of state that predominantly determines the protoneutron star evolution.

Keywords: equation of state, protoneutron stars, gravitational waves

1. Introduction

The gravitational waves from the mergers of binary black holes and a binary neutron star have been successfully detected[1,2], which leads to the beginning of a new age of the gravitational wave astronomy. In addition to the second-generation detecters, such as Advanced LIGO, Advanced VIRGO, and KAGRA, third-generation detecters like Einstein Telescope and Cosmic Explorer are already being proposed. Via observations with such detectors, one would obtain an imprint of open problems in compact objects. The most promising source for gravitational waves must be a merger of binary system of compact objects, while the core-collapse supernovae, which produce compact objects, could be a secondary candidate.

Up to now, in order to study the gravitational wave signals from core-collapse supernovae, extensive numerical simulations have been done. Through the results obtained by such simulations, it is considered that the g-mode oscillations excited around the protoneutron star surface are one of the most important gravitational wave emission in the postbounce phase[3–5]. In practice, the typical frequency of g-mode oscillations is expressed as $\sim M_{\rm PNS}/R_{\rm PNS}^2$ with the mass $M_{\rm PNS}$ and radius $R_{\rm PNS}$ of a protoneutron star. Thus, one may extract the information of protoneutron star properties by observing the g-mode gravitational waves from protoneutron stars.

On the other hand, as another approach, the gravitational-wave asteroseismology is a powerful technique for extracting interior information of compact objects. With such a technique one can constrain the equation of state (EOS), mass, and compactness of cold neutron stars (e.g., Refs. 6, 7, 8). With respect to the case of protoneutron stars, a few studies have been done[9–12]. In this study, by adopting the results of numerical simulation of relativistic three-dimensional supernova models, we systematically examine the spacetime oscillations, the so-called w-modes, of gravitational waves from protoneutron stars. More details about this study can be seen in Ref. 10.

2. Protoneutron star models

To make a linear analysis, one has to prepare background protoneutron star models. In the present study, we adopt the numerical results of relativistic three-dimensional simulation[5] with the $15M_\odot$ progenitor models, adopting two different EOSs, i.e., SFHx and TM1. In particular, we consider very early phase up to ~ 250 ms after core-bounce. In order to provide a static, spherically symmetric background model, the numerical data on each time step is averaged in the angular direction. Then, the protoneutron star's surface is determined at a fiducial rest-mass density of 10^{10} g/cm^3. In the left panel of Fig. 1, the evolution of protoneutron star models with two EOSs is shown in $M_{\rm PNS}$–$R_{\rm PNS}$ plane. With time after core-bounce, the mass increases due to the mass accretion, while the radius decreases due to the cooling. So, the mass and radius of protoneutron stars change with time from lower right to upper left in the left panel of Fig. 1. We remark that the sequence of mass and radius of protoneutron stars depends strongly on the adopted EOS.

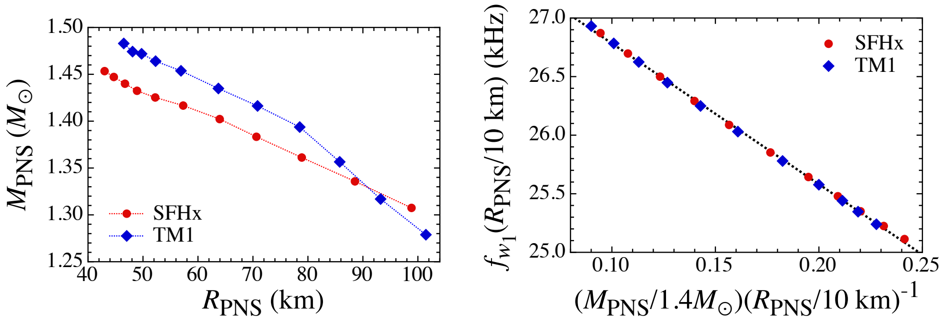

Fig. 1. Evolution of mass and radius of protoneutron stars in the left panel, and the w_1-mode multiplied by stellar radius as a function of stellar compactness in the right panel[10]. The circles and diamonds correspond to the results with SFHx and TM1 EOSs.

3. Asteroseismology in protoneutron stars

The perturbation equation derived from the linearized Einstein equations should be solved together with appropriate boundary conditions. First, we focus on the spacetime oscillations, the so-called w-modes, of gravitational waves from protoneutron star models. Since this mode is an oscillations of spacetime itself, the compactness, which is the ratio of $M_{\rm PNS}$ to $R_{\rm PNS}$, is an important parameter for describing the frequencies[6]. In practice, we calculate the w-mode frequencies with the protoneutron star models at each time step and find that the evolution of frequencies depends on the adopted EOS. However, we also find that the frequencies multiplied with the radius can be expressed well as a linear function of the stellar compactness independently of the adopted EOS, as shown in the right panel of Fig. 1. In fact, we can derive the fitting formula of the w_1-mode frequencies as

$$f_{w_1}^{(\mathrm{PNS})}(\mathrm{kHz}) \approx \left[27.99 - 12.02\left(\frac{M_{\mathrm{PNS}}}{1.4M_\odot}\right)\left(\frac{R_{\mathrm{PNS}}}{10\ \mathrm{km}}\right)^{-1}\right]\left(\frac{R_{\mathrm{PNS}}}{10\ \mathrm{km}}\right)^{-1}, \tag{1}$$

which is also plotted in the right panel of Fig. 1 with the dotted line.

On the other hand, we calculate the f-mode gravitational waves as in Ref. 9. Since the f-mode oscillations are a kind of acoustic oscillations, the frequencies can be characterized by the sound velocity or the stellar average density $M_{\mathrm{PNS}}/R_{\mathrm{PNS}}^3$[6]. In fact, we find that the time evolution of f-mode frequencies from protoneutron star depends on the adopted EOS, but the frequencies can be expressed as a linear function of the square root of the stellar average density independently of the adopted EOS as

$$f_f^{\mathrm{PNS}}(\mathrm{Hz}) \approx 14.48 + 4859\left(\frac{M_{\mathrm{PNS}}}{1.4M_\odot}\right)^{1/2}\left(\frac{R_{\mathrm{PNS}}}{10\ \mathrm{km}}\right)^{-3/2}. \tag{2}$$

Now, we have two formulae for describing the f- and w_1-mode gravitational waves from protoneutron stars, which are independent of the adopted EOS for dense matter. So, by simultaneously observing the time evolution of two modes in the gravitational waves, one can obtain the time evolution of the stellar average density and compactness separately. That is, one can determine the mass and radius of protoneutron star at each time step after core-bounce as shown in the left panel of Fig. 1, which provides a new probe into the finite-temperature nuclear EOS.

4. Conclusion

The gravitational waves from supernovae are one of the most promising source. In the present analysis, we systematically examine the w_1- and f-mode gravitational waves after core-bounce, using the data of numerical simulations of relativistic three-dimensional supernova as a background model, and successfully derive universal formulae of w_1- and f-mode oscillations as a function of stellar compactness and average density independently of the adopted EOS. Thus, with the direct observations of gravitational waves from core-collapse supernovae, if one can identify the time evolution of f- and w_1-mode oscillations, one can in turn see the evolution of stellar average density and compactness, which tells us the evolution of mass and radius of a protoneutron star. As a result, one can in principle determine the finite-temperature nuclear EOS even through one event of gravitational waves from the core-collapse supernova.

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