Primordial black hole formation in inflation models and production of gravitational waves

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Primordial black holes (PBHs) are one of promising candidates to explain the gravitational wave events observed by the LIGO detectors. PBHs are produced from large density perturbations generated during inflation. We show that a multi-field inflation model can produce a sufficient number of PBHs. In particular, the model predicts two peaks in PBH mass spectrum and hence can explain not only LIGO GW events but also all dark matter of the universe. Furthermore, density perturbations large enough to produce PBHs generate gravitational waves via the second-order effect. The produced gravitational waves have a broad spectrum (nHz - O(10)Hz) and can be detected in future GW detectors like SKA, LISA and DECIGO.

Keywords: Primordial black hole; Gravitatinal waves; Inflation model

1. Introduction

In 2015, the Advanced LIGO detected gravitational waves (GWs) directly for the first time[1]. This event, GW150914, comes from the merger of black holes (BHs) with $\sim 30M_\odot$. After the first detection, two more GW events caused by the merger of BHs with similar masses were reported[2,3]. One of the promising candidates for BHs causing the LIGO GW events is primordial black holes (PBHs).

PBHs are BHs formed in the early universe by the gravitational collapse of the over-dense regions[4–6]. PBHs also have been considered as dark matter (DM) candidate. Although several observations have ruled out most of the region for PBH dark DM, PBHs with mass around 10^{-14}–$10^{-10}M_\odot$ can still explain all DM.

Inflation can generates large density perturbations required to produce PBHs. However, the amplitude of the density perturbations is small ($\sim 10^{-5}$) on large scales. In order to produce PBHs the density perturbations are very large (~ 0.1) on small scales, so we need to break the scale invariance of the curavture perturbations, which is hard to realize in single-field inflation models. One way to overcome this difficulty is to consider multi-field inflation models. Here we adopt the double inflation model where two stages of inflation (preinflation and new inflation) take place[7–9]. In particular, the model predicts two peaks in PBH mass spectrum and hence can explain not only LIGO GW events but also all DM of the universe.

Since large curvature perturbations are required for PBH formation, the second-order effect of the curvature perturbations cannot be neglected. In fact, they give a contribution to the source term in the equation of motion for GWs and a significant amount of GWs is produced. The double inflation model predicts GWs with frequency from nHz to $\mathcal{O}(10)$Hz which can be detected by future GW detectors.

2. PBH formation

PBHs are formed when an overdense region with density perturbation larger than the critical value $\delta_c \simeq 0.4$ reenter the horizon. The mass of a PBH is roughly given by the horizon mass at the horizon entry of the perturbation. The relation between the scale of the perturbation and the PBH mass is

$$M \simeq M_\odot \left(\frac{\gamma}{0.2}\right)\left(\frac{g_*}{10.75}\right)^{-\frac{1}{6}}\left(\frac{k}{1.9\times 10^6\,\mathrm{Mpc}^{-1}}\right)^{-2}, \tag{1}$$

where γ is the fraction of the PBH mass in the horizon mass at the formation and depends on the detail of the gravitational collapse.

The production rate per logarithmic Hubble time interval is estimated using the Press-Schechter formalism as

$$\beta(M) = \int_{\delta_c} \frac{d\delta}{\sqrt{2\pi\sigma^2(M)}}\, e^{-\frac{\delta^2}{2\sigma^2(M)}} \simeq \frac{1}{\sqrt{2\pi}}\frac{1}{\delta_c/\sigma(M)}\, e^{-\frac{\delta_c^2}{2\sigma^2(M)}}, \tag{2}$$

where $\sigma^2(M)$ is the coarse-grained density contrast with the smoothing scale k. The current fraction of dark matter(DM) in PBHsis given by

$$f(M) \simeq \left(\frac{\beta(M)}{1.84\times 10^{-8}}\right)\left(\frac{\gamma}{0.2}\right)^{\frac{3}{2}}\left(\frac{10.75}{g_*}\right)^{\frac{1}{4}}\left(\frac{0.12}{\Omega_{\rm DM}h^2}\right)\left(\frac{M}{M_\odot}\right)^{-\frac{1}{2}}, \tag{3}$$

where $f(M) \equiv \frac{1}{\Omega_{\rm DM}}\frac{d\,\Omega_{\rm PBH}}{d\ln M}$.

3. Double inflation

The double inflation model consists of preinflation and new inflation. The preinflation generates curvature perturbations on large scales observed by CMB while the new inflation generates large density perturbations on small scales, which leads to PBH formation. The potential of the new inflation is written as

$$V_{\rm new}(\varphi) = -cv^2\varphi - \kappa v^4\varphi^2 + \left(v^2 - g\varphi^3\right)^2, \tag{4}$$

where φ is the inflaton, v is the new inflation scale and c, κ, g are constants ($M_{pl} = 1$).

For PBHs to explain the DM and the LIGO events simultaneously, the PBH mass spectrum must have peaks around $\mathcal{O}(10^{13})M_\odot$ and $\mathcal{O}(10)M_\odot$. This means that the power spectrum of the curvature perturbations must have peaks at $\mathcal{O}(10^{12})\mathrm{Mpc}^{-1}$ and $\mathcal{O}(10^6)\mathrm{Mpc}^{-1}$. In the double inflation model, the enhancement of the perturbations can be realized by two mechanisms. One is the enhancement due to the inflection point of the new inflation potential. Another mechanism for enhancement is due to dynamics of the inflaton field φ during oscillating phase after preinflation. If the effective mass of φ is smaller than the Hubble the fluctuations of φ generated during preinflation reenter the horizon without damping at the beginning of new inflation and enhance the curvature perturbations. Thus, correspondingly to the two mechanism, the double inflation can produce two peaks in the power spectrum

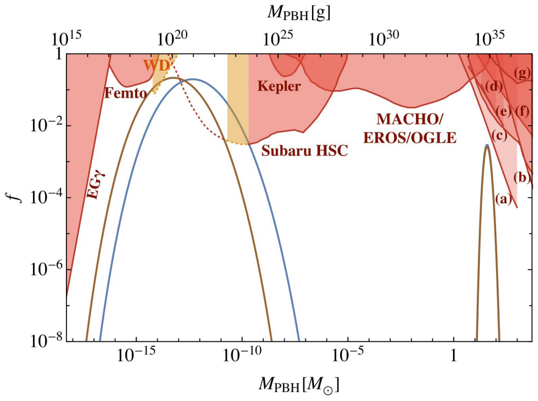

Fig. 1. PBH mass spectra for two appropriate set of the model parameters (blue and brown lines)[10]. The shaded regions denote the observational constraints.

of the curvature perturbations and hence two peaks in the PBH mass spectrum as shown in Fig. 1

4. Gravitational waves

The spectrum of the induced GWs in the double inflation model is shown in Fig. 2. The spectrum has two bumps corresponding to the two peaks in the power spectrum of the curvature perturbations. The sharp peak around nHz corresponds to $\mathcal{O}(10)_\odot$ PBHs, which avoid the current constraint from pulsar timing array experiments[11–13]. The spectrum at lower frequencies has a broad peak which comes from the curvature perturbations responsible for DM PBHs with mass $\mathcal{O}(10^{-13})M_\odot$. The predicted GW spectrum is above the sensitivity of SKA[14,15], (e)LISA[16,17] and DECIGO[18]. One can see that both bumps corresponding to PBHs for DM and LIGO will be probed by planned observations of GWs.

5. Conclusion

It is found that the double inflation model can simultaneously explain both LIGO PBHs and DM PBHs. The large curvature perturbations required for PBH formation also generate GWs as the second-order effect. The GWs produced in the double inflation model can be detected by future GW detectors and hence the model can be tested.

References

1. B. P. Abbott *et al.* [LIGO Scientific and Virgo Collaborations], Phys. Rev. Lett. **116**, no. 6, 061102 (2016) [arXiv:1602.03837 [gr-qc]].
2. B. P. Abbott *et al.* [LIGO Scientific and VIRGO Collaborations], Phys. Rev. Lett. **118**, no. 22, 221101 (2017) [arXiv:1706.01812 [gr-qc]].

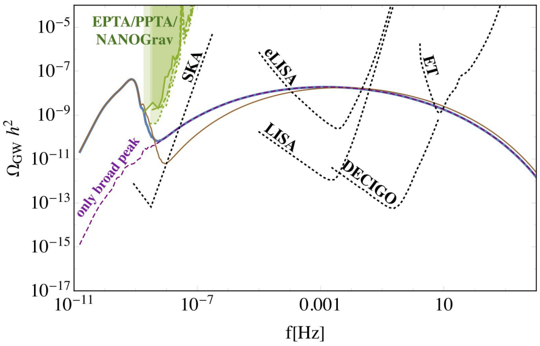

Fig. 2. Gravitational wave spectra for two set of the model parameters. The shaded region is excluded by the pulsar timing array experiments. Black dotted lines show the prospects of the future experiments.

3. B. P. Abbott *et al.* [LIGO Scientific and Virgo Collaborations], Phys. Rev. Lett. **119**, no. 14, 141101 (2017) [arXiv:1709.09660 [gr-qc]].
4. S. Hawking, Mon. Not. Roy. Astron. Soc. **152**, 75 (1971).
5. B. J. Carr and S. W. Hawking, Mon. Not. Roy. Astron. Soc. **168**, 399 (1974).
6. B. J. Carr, Astrophys. J. **201**, 1 (1975).
7. M. Kawasaki, N. Sugiyama and T. Yanagida, Phys. Rev. D **57**, 6050 (1998) [hep-ph/9710259].
8. M. Kawasaki, A. Kusenko, Y. Tada and T. T. Yanagida, Phys. Rev. D **94**, no. 8, 083523 (2016) [arXiv:1606.07631 [astro-ph.CO]].
9. K. Inomata, M. Kawasaki, K. Mukaida, Y. Tada and T. T. Yanagida, Phys. Rev. D **95**, no. 12, 123510 (2017) [arXiv:1611.06130 [astro-ph.CO]].
10. K. Inomata, M. Kawasaki, K. Mukaida and T. T. Yanagida, Phys. Rev. D **97**, no. 4, 043514 (2018) [arXiv:1711.06129 [astro-ph.CO]].
11. L. Lentati *et al.*, Mon. Not. Roy. Astron. Soc. **453**, no. 3, 2576 (2015) [arXiv:1504.03692 [astro-ph.CO]].
12. R. M. Shannon *et al.*, Science **349**, no. 6255, 1522 (2015) [arXiv:1509.07320 [astro-ph.CO]].
13. Z. Arzoumanian *et al.* [NANOGrav Collaboration], Astrophys. J. **821**, no. 1, 13 (2016) [arXiv:1508.03024 [astro-ph.GA]].
14. C. J. Moore, R. H. Cole and C. P. L. Berry, Class. Quant. Grav. **32**, no. 1, 015014 (2015) [arXiv:1408.0740 [gr-qc]].
15. G. Janssen *et al.*, PoS AASKA **14**, 037 (2015) [arXiv:1501.00127 [astro-ph.IM]].
16. B. S. Sathyaprakash and B. F. Schutz, Living Rev. Rel. **12**, 2 (2009) [arXiv:0903.0338 [gr-qc]].
17. P. Amaro-Seoane *et al.*, GW Notes **6**, 4 (2013) [arXiv:1201.3621 [astro-ph.CO]].
18. K. Yagi and N. Seto, Phys. Rev. D **83**, 044011 (2011) Erratum: [Phys. Rev. D **95**, no. 10, 109901 (2017)] [arXiv:1101.3940 [astro-ph.CO]].