Gravitational-wave detector using optical lattice clocks in space

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The new technique of measuring time by optical lattice clocks now approaches at the level of $\Delta t/t = 10^{-18}$. We propose to place such precise clocks in space and to use Doppler tracking method for detecting low-frequency gravitational wave. Our plan is to locate three satellites at one A.U. distance (say at L1, L4 & L5 of Sun-Earth orbit), and to apply reachable current technologies, then we obtain three or four order improvement (10^{-17} level in 10^{-5}Hz – 1 Hz) than the sensitivity of Cassini satellite in 2001. This sensitivity enables to search inspirals of inter-mediate black holes in *Gpc* order. Based on the hierarchical growth model of black-holes in galaxies, we estimate the event rate of detections will be 20-30 for a year.

Keywords: Gravitational Waves, Detector proposals in Space, Super-massive black-holes

Evolution process of black holes revealed via gravitational waves

Almost all of the galaxies in the Universe have super-massive black holes (SMBHs) in their center, which mass is over $10^6 M_\odot$ [1]. Observational data show that the masses of such SMBHs are proportional to the masses of their bulge (the center part of the galaxies) [2,3]. This fact indicates that the SMBHs in the galaxies coevolved with its mother galaxy, but such an inevitable relation is still a mystery in the history of the Universe.

One of the plausible scenario is the hierarchical growth model of the stars in the galaxy. This model says that a black hole (a seed black hole) is first formed in the center of galaxy when it grows to the size of a dwarf galaxy. The intermediate-mass black holes (IMBHs), which mass is around 100∼1000 $M_\odot$, in its star clusters will then accumulate in the center region of the galaxy, merge together, and form to a SMBH [4]. A seed black hole might be formed by another process; one possible way is to start from a forming a $10^5 M_\odot$ black hole in the early stage of the Universe. Hierarchical mergers will proceed between galaxies, and such processes will naturally produce SMBHs [5,6]. In result, the mergers of $10^6 - 10^{10} M_\odot$ black holes are expected to be the sources of gravitational wave [7,8].

Formation of SMBHs are also modeled by accumulations of large amount of gas to a seed BH [9]. Therefore, collecting the detections of gravitational waves from large mass BHs, which means the direct evidence of the evolution process of the BHs, is the clue to solve the current mystery of coevolution of a BH and its mother galaxy.

All the gravitational detectors which are under operation today, such as LIGO and Virgo, are located on the ground, which means that we are hard to detect gravitational wave below 10 Hz since seismic vibration dominates as noise. The mergers of BHs larger than $10^4 M_\odot$, on the other hand, end up below 1 Hz. Therefore the only possible way to detect such a gravitational wave signal is to settle detectors in space. The project of eLISA satellites by ESA[10], and the proposed project B-DECIGO satellites in Japan[11] are the space-borne interferometers for detecting gravitational wave which are planned to be launched in 2030s $\sim$ 40s.

Space-borne interferometers such as eLISA or B-DECIGO require technical breakthroughs in future. We, in this article, propose an alternative method for detecting low-frequency gravitational waves, although the sensitivity is weak but technically feasible within today's our knowledge.

Gravitational wave detection using optical lattice clocks

The only observations of gravitational wave in space so far are the one by tracking artificial satellite using Doppler effect[12] (actually they showed us the upper-bound constraint of the gravitational wave). The method is to observe the velocity shift (Doppler shift) produced by passing gravitational wave between the Earth and the satellite, by comparing the frequency of the signal sent from the Earth and received at the satellite using their clocks. The sensitivity of this Doppler-tracking method depends on the distance of the signal baseline. Until now, the most strict sensitivity was obtained by the Cassini satellite which was launched for surveying Saturn[13].

The key technology of this Doppler-tracking method is the stability of clocks. In these years, we have tremendous developments in this field; the optical lattice clock developed by Katori *et al* had obtained the accuracy of 10^{-18}[14] and will approach to 10^{-19} near future. (Note that the accuracy of our standard Cs atomic clock is 10^{-16}). If this level of the accuracy is obtained, then the noise from clock can be completely ignored.

Kolkowitz *et al*[15] proposed to use optical lattice clocks to detect gravitational waves. Their idea is to measure the Doppler shift between the two optical lattice clocks in space which are communicating with lasers. By controlling two mirrors located apart in the drag-free state, they propose to measure the frequency difference between two optical lattice clocks using precise laser which are linked with these mirrors. The core idea is the same with that of the Doppler-tracking method, but with the technology of drag-free control, they say the sensitivity is greatly improved at 0.01$\sim$1 Hz. However, there is a disadvantage in drag-free technology. The remained acceleration of free mass is controlled with magnetic field, but when cosmic-ray hits the device, the photoelectric effect charges the free-mass and this fluctuation behaves noise. Especially at the lower frequency, the residual error is inversely proportional to the square of frequency[16], and as a result the sensitivity of their proposal is at the same level with eLISA.

We therefore propose not to use drag-free control, but to improve Doppler-tracking method with advanced optical lattice clock and the light-linking technology for constructing a gravitational wave detector.

Feasibility of the satellites

We estimate the reachable sensitivity for gravitational wave detection with current known technologies. In order to make the most feasible discussion, we do not consider to use drag-free control, nor precise laser control, but simply apply the advanced optical lattice clock to the Doppler-tracking method.

As we already mentioned, it is preferable to locate the satellites for Doppler tracking at far distance such as beyond the orbit of Jupiter or Saturn. This request, however, is severe for keeping power and fuel. We therefore propose to locate three satellites at the Lagrangian points (L1, L4, L5) of the Sun-Earth orbit, which enable us to take the baselines between each satellites at the order of A.U. We also propose to communicate them with radio or light. If we link them with radio, then double tracking method which uses two-frequency bands will compensate the phase shift due to interplanetary plasma. If we link them with light, there is no fluctuations by such plasma.

The sensitivity of the Doppler-tracking method is well understood by the report of Cassini satellite[13,16], which keeps the best record as $h_{\rm n} \sim 3 \times 10^{-15}$ at 10^{-4} Hz, where $h_{\rm n}$ is the noise amplitude, which is given by the square root of the combination of the power spectrum of the noise times frequency f. The noise amplitude is the standard quantity since it can be compared directly with the characteristic strain $h_{\rm c}$ which expresses the strength of the gravitational wave signal. Cassini's sensitivity showed the curve f^{-1} below 10^{-4} Hz.

We here consider the four-order improved version of Cassini satellite (i.e. the minimum sensitivity is around $h_{\rm n} = 10^{-18}$). The origins of noise in Cassini are identified mostly from the accuracy of the atomic clock and from the fluctuation of troposphere of the Earth[13]. If we use the advanced optical lattice clock instead of the atomic clock, and let the satellites communicate each other directly, then the sensitivity will be dramatically improved. If we had the baseline of 1.5×10^8 km ($\sim$ 1 A.U.), then the sensitivity of $h_{\rm n} = 10^{-17}$ is comparable with the amplitude 0.7 mm; which corresponds to the measurement of the velocity 2×10^{-8} m s^{-1}, or the acceleration 1.4×10^{-12} m s^{-2} at 10^{-5} Hz. If we can ignore the noise both from the clock and from the troposphere of the Earth, then the sensitivity at the lower frequency is dominated by the velocity fluctuation due to the radiation pressure of the Sun beam. The radiation pressure force is $F = P/c$, where P is the power of the Sun beam and c is the speed of light. Around the Earth orbit, P is 1.3 kW m^{-2} per unit area. If we suppose the solar cell panel as 10 m^2 and the mass of the satellite as 1000 kg, then the acceleration of the satellite due to the beam pressure is about 5×10^{-8} m s^{-2}. The pressure of the Sun fluctuates at the order of 10^{-3}, so that the acceleration fluctuates at the order of 10^{-11}. In order to detect the gravitational wave of which acceleration is at the order of 10^{-12}, we should reduce the fluctuation one order smaller. This is attainable by separating the satellite from its solar cell panel and remove their dynamical interactions which may reduce the effects of the Sun beam two-orders of magnitude.

Expected gravitational wave events

Fig. 1 shows the sensitivity curve of Cassini satellite and their 1∼4-order improved version (we named Cassini+, Cassini++ and so on), together with that of eLISA. In Fig. 1, we also plot the characteristic strain of the gravitational wave (h_c) from a merger of the binary BHs with its distance 1 Gpc from the Earth. We plotted for mergers of equal-mass BHs for several different masses. Each line starts from its frequency when the binary's separation is 50 times of their event horizon radius, and ends at the frequency when they merge. We see that the mergers of SMBHs of $10^7 \sim 10^8 M_\odot$ produce gravitational wave around 10^{-4} Hz, which is detectable with Cassini+++ at the signal-to-noise ratio (SNR) 10.

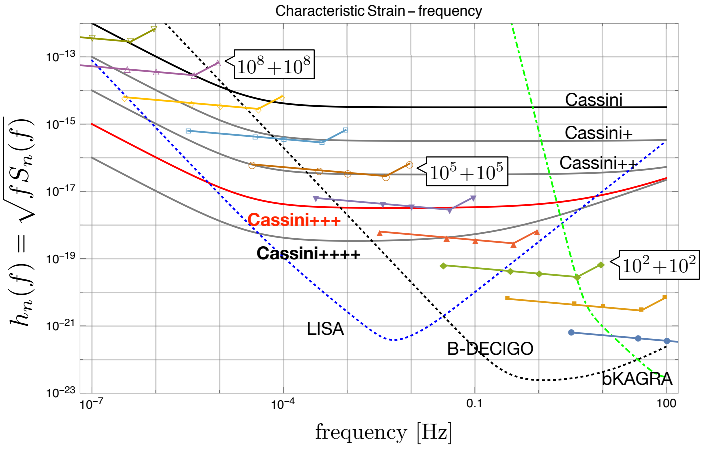

Fig. 1. Sensitivity of Doppler-tracking satellites and expected strains of gravitational wave. The most upper solid curves indicates the sensitivity of Cassini satellite, while the other solid curves are those of 1∼4-order improved version (we named Cassini+, Cassini++ and so on). The dotted line is the sensitivity curve of eLISA. Almost horizontal lines with symbols indicate the characteristic strain of gravitational wave from a merger of equal-mass binary BHs at 1 Gpc. Each line is for their inspiral phase; starts from their separation is 50 times of their event horizon radius, and ends at their merger. (Frequency moves up higher for smaller separation).

In Fig. 2, we plot the detectable distance (observational distance, or horizon of the detector) of the satellites. We see Cassini+++ already covers the Universe for the mergers of SMBHs of their chirp mass $10^7 \sim 10^8 M_\odot$. (For the binary of masses m_1 and m_2, the chirp mass, M_c, is given by $M_c = (m_1 m_2)^{3/5}/(m_1+m_2)^{1/5}$. M_c determines the leading-order amplitude and frequency evolution of the gravitational-wave signal from inspiraling binary.)

If we further assume the distribution model of BH mass (i.e. the evolution model of BHs), and the distribution model of galaxies, together with cosmological model, then we can estimate the event rate of gravitational wave per year. Applying the model by Shinkai *et al*[8], Cassini++++ may detect 30 events per year for inspiral

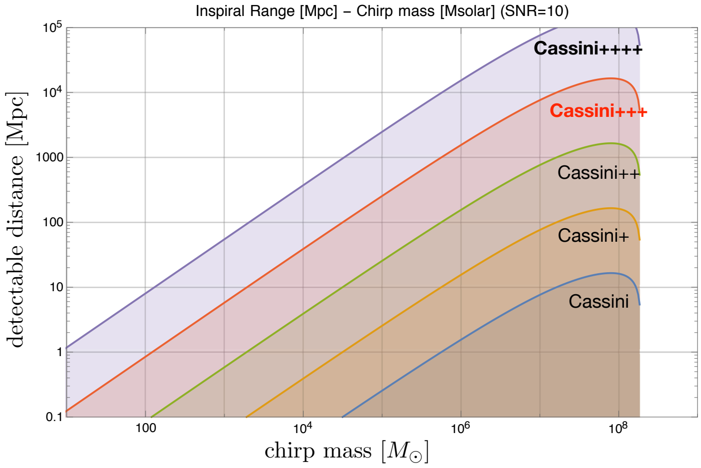

Fig. 2. Detectable distance (observational distance, or horizon of the detector) of Cassini, $\cdots$, Cassini++++ as a function of the binary's chirp mass. The distance is the luminosity distance. All lines are for SNR=10.

phase of BH binary for their chirp mass $> 10^3 M_\odot$, which we think worth trying to detect.

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