Is $E=\Delta mc^2$ is mathematically derived or speculated in Sep.1905 paper?

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Abstract

In this paper Einstein derived $\Delta L=\Delta mc^2$ (light energy –mass equation), it is not completely studied; and is only valid under special conditions of involved parameters e.g. number of light waves, magnitude of light energy, angles at which waves are emitted and relative velocity v. Einstein considered just two light waves of equal energy, emitted in opposite directions and velocity v is uniform. There are numerous possibilities of parameters which are not considered in Einstein's derivation. $\Delta E=\Delta mc^2$ is obtained from $\Delta L=\Delta mc^2$ by simply replacing L by E (every energy) without derivation. Fadner pointed out that Einstein neither mentioned E or $\Delta E=\Delta mc^2$, in the derivation which is absolutely correct. Here results are critically analysed taking all possible variables in account. Under some conditions of valid parameters $\Delta L=\Delta mc^2$ is not obtained e.g. sometimes result is $M_a=M_b$ or no equation is derivable. If all values of valid parameters are taken in account then the same derivation also gives $L \alpha \Delta mc^2$ or $L=A\,\Delta mc^2$, where A is coefficient of proportionality. Thus Einstein's derivation under the valid parameters also predicts that energy emitted may be less or more than $\Delta L=\Delta mc^2$.

1.0 Description and critical analysis of Einstein's Thought Experiment

In Einstein's derivation basic equation is

$$\lambda^* = \lambda \frac{\left[1 - \frac{v}{c}\cos\phi\right]}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (1)$$

where λ is light energy emitted by body in frame (x,y,z) and λ^* is light energy measured in system (ξ, η, ζ), and v is velocity with which the frame or system (ξ, η, ζ) is moving. This is equation for Doppler principle for light for any velocity whatever [1]. The eq.(1) is based upon constancy of speed of light and c is maximum limit for speed of any particle.

Einstein's perception [1]: Let a system of plane waves of light, referred to the system of coordinates (x, y, z), possesses the energy l; let the direction of the ray (the wave-normal) makes an angle ϕ with the axis of x of the system [2]. Energy is a scalar quantity, having magnitude only, but according to eq.(1) it depends upon angle also.

If we introduce a new system of co-ordinates (ξ, η, ζ) moving in uniform parallel translation with respect to the system (x, y, z), and having its origin of coordinates in motion along the axis of x with the velocity v.

Thus v is the relative velocity between system (x, y, z) and system (ξ, η, ζ). The body which emits light energy is considered stationary in the system (x,y,z) and also remains stationary after emission of light energy in the system (xy,z).

Let E_0 and H_0 are energies in coordinate system (x, y, z) and system (ξ, η, ζ) before emission of light energy, further E_1 and H_1 are the energies of body in the both systems after it emits light energy. E_i and H_i include all the energies possessed by body in two systems. The various meanings of E_i's and H_i's are shown in Table I.

Table I. Energies emitted before and after emission by body in Einstein's Sep. 1905 derivation.

Sr No	System (x,y,z) at rest	System(ξ, η, ζ) moving with velocity v
1	Before Emission E_0	Before Emission H_0
2	After Emission E_1	After Emission H_1

Then Einstein concluded that body emits two light waves of energy 0.5L each in system (x,y,z) where energy is E_0. Thus,

Energy before Emission = Energy after emission +0.5L + 0.5L

$$E_0 = E_1 + 0.5L + 0.5L = E_1 + L \quad (2)$$

Energy of body in system (ξ, η, ζ)

$$H_0 = H_1 + 0.5\ \beta\ L\{(1 - \frac{v}{c}\cos\phi) + (1 + \frac{v}{c}\cos\phi)\} \quad (3)$$

where $\beta = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$ (4)

$$H_0 = H_1 + \beta L \quad (5)$$

Or $(H_0 - E_0) - (H_1 - E_1) = L\ [\beta - 1]$ (6)

Einstein maintained as

$$(H_0 - E_0) = K_0 + C = \frac{M_b v^2}{2} + C$$

$$(H_1 - E_1) = K_1 + C = \frac{M_a v^2}{2} + C$$

Einstein defined C as additive constant which depends on the choice of the arbitrary additive constants of the energies H and E. The arbitrary additive constant C is regarded as equal in both the cases. Kinetic energy of body before emission of light energy, K_0 ($\dfrac{M_b v^2}{2}$) and kinetic energy of body after emission of light energy, K_1 ($\dfrac{M_a v^2}{2}$).

$$K_0 - K_1 = L\ \{\frac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} - 1\} \quad (7)$$

Einstein considered the velocity in classical region thus applying binomial theorem,

$$K_0 - K_1 = L\left(1 + \frac{v^2}{2c^2} + 3\frac{v^4}{8c^4} + ... - 1\right) \quad (8)$$

Further Einstein quoted [2]

Neglecting magnitudes of fourth (v^4/c^4) and higher(v^6/c^6, v^8/c^8 …….) orders, we may place.

$$K_0 - K_1 = L\ \frac{v^2}{2c^2} \quad (9)$$

$$\frac{M_b v^2}{2} - \frac{M_a v^2}{2} = L\ \frac{v^2}{2c^2} \quad (10)$$

or $L = (M_b - M_a)\ c^2 = \Delta m c^2$ (11)

or Mass of body after emission (M_a) = Mass of body before emission (M_b) – $\frac{L}{c^2}$. (12)

Then Einstein generalized the result for every energy and called mass of body is measure of energy content (every energy that is included in a collection). Fadner [3] has mentioned that in the paper Einstein neither wrote E= Δmc^2 nor E in the paper. It is concluded that Einstein's statement means E= Δmc^2 **.** So it is speculation. It can be obtained by replacing L (light energy) by E (energy-content or every energy). Einstein wrote,

$$E = (M_b - M_a) c^2 = \Delta mc^2 \quad (13)$$

or Mass of body after emission (M_a) = Mass of body before emission (M_b) – $\frac{E}{c^2}$ (14)

When energy is emitted the mass decreases. Thus Einstein did not differentiate between Light Energy and other energies in the derivation.

1.1 Typical comments regarding classical region of velocity (not given by Einstein).

Einstein's derivation also offers the most mysterious situation in science. It is explained below, it is explained below with help of relativistic variation of mass equation [4-8],

$$M_{motion} = \frac{M_{rest}}{\sqrt{1-\frac{v^2}{c^2}}} \quad (15)$$

Let the velocity is in classical region i.e. 1m/s (3.6 km/hr i.e. ordinary speed of vehicle), then no increase in mass of object when it moves with this velocity. The speed of aeroplane is over 400km/hr, and no increase in mass is observed.

$M_{motion} = M_{rest} [1+ \mathbf{v^2/2c^2} + 3v^4/8c^4 + \ldots\ldots\ldots\ldots] \quad (16)$

(i) If v =0 , then

$M_{motion} = M_{rest}$

(ii) If v = 1cm/s (0.036 km/hr)

$M_{motion} = M_{rest} [1+ \mathbf{5.55\times10^{-22}} + \mathbf{4.166\times10^{-42}} + \ldots\ldots\ldots\ldots..] \quad (17)$

$M_{motion} = M_{rest} + M_{rest}\ \mathbf{5.55\times10^{-22}} + M_{rest}\ \mathbf{4.166\times10^{-42}} + \ldots\ldots\ldots$

Here even term $\mathbf{5.55\times10^{-22}}$ is regarded as negligible compared to unity , and $\mathbf{4.166\times10^{-42}}$ is further negligible thus

$$M_{motion} = M_{rest} \quad (18)$$

Thus term 5.55×10^{-22} can be neglected only then both masses are equal.

(iii) Similarly the orbital velocity of the earth is 30km/s or 3,0000m/s i.e. v/c =10^{-4} thus

$M_{motion} = M_{rest} [1+ \mathbf{v^2/2c^2} + 3v^4/8c^4 + \ldots\ldots\ldots\ldots]$

$= M_{rest} [1+ \mathbf{5\times10^{-9}} + 3.75\times10^{-17} + \ldots\ldots\ldots\ldots..]$

$\mathbf{M_{motion} = M_{rest} + M_{rest}\ 5\times10^{-9} + M_{rest}\ 3.75\times10^{-17}}$ (19)

The mass of earth remains same i.e. 5.98×10^{24} kg always. Thus here also the term $v^2/2c^2$ (5×10^{-9}) is neglected compared to unity. If the term $v^2/2c^2$ $(\mathbf{5\times10^{-9}})$ is neglected then

M_{motion} [mass of earth in motion] = M_{rest} [mass of earth at rest] (20)

The various terms neglected compared to unity are shown in Table II

Table II: Terms neglected in calculations and their effects.

Sr. No.	velocity	$M_{rel} = M_{rest}[1+v^2/2c^2+3v^4/8v^4 +\ldots.]$	Neglected term	Result
1	0	$M_{rel} = M_{rest}$	none	$M_{rel} = M_{rest}$
2	Earth's orbital velocity 30km/s or 3×10^4m/s	$M_{rel} = M_{rest} [1+ \mathbf{5\times10^{-9}} + 3.75\times10^{-17} + \ldots\ldots\ldots]$	$\mathbf{5\times10^{-9}}$	$M_{rel} = M_{rest}$
3	v=1cm/s or 0.036km/s	$K_b - K_a$ $= L [1+ \mathbf{5.55\times10^{-22}} +4.166\times10^{-42} + \ldots\ldots\ldots-1]$ or $M_b = M_a$	$\mathbf{5.55\times10^{-22}}$	$M_b = M_a$ Mass before emission = Mass after Emission

1.2 Appearance of c^2 in $\Delta L = \Delta mc^2$ is apparently arbitrary.

$$K_0 - K = L\left(1+\frac{v^2}{2c^2}+3\frac{v^4}{8c^4}+\ldots\ldots\ldots\ldots\ldots\ldots-1\right) \quad (8)$$

Now consider the same case when velocity is 1cm/s or 0.036km/hr , under this conditions eq.(8) becomes

$$\frac{M_b v^2}{2}-\frac{M_a v^2}{2} = L [1+ \mathbf{5.55\times10^{-22}} \mathbf{+4.166\times10^{-42}} + \ldots\ldots\ldots..-1] \quad (21)$$

(i) Einstein has neglected term $3v^4/8c^4$ retained the term as $v^2/2c^2$, and obtained equation

$$L = \Delta mc^2$$

(ii) If the velocity is very-2 small then $v^2/2c^2$ can be neglected compared to unity. If velocity is 1cm/s (classical region), then $v^2/2c^2$ is 5.55×10^{-22}. Depending upon the orbital velocity of the earth (30km/s or 3,0000m/s i.e. v/c =10^{-4}) the term $\mathbf{v^2/2c^2}$ **(5×10^{-9}**) can be neglected compared to unity, only then the equation i.e

M_{motion} [mass of earth in motion] = M_{rest} [mass of earth at rest]

is justified.

In typical classical region (v =1cm/s) $\mathbf{v^2/2c^2}$ **=** 5.55×10^{-22} is neglected compared to unity (as **5×10^{-9}** is neglected) then

M_b (mass before emission) = M_a (mass after emission) (22)

Thus both $\Delta L = \Delta mc^2$ and $M_b = M_a$ are equally probable and but have entirely different nature. This discussion also validates the necessity of categorisation of sub ranges of velocity in the classical region or up to which magnitude of the term to be neglected comparatively.

2.0 Einstein took only *super special* values of variables and its effects.

The following arguments can be given that Einstein's derivation is true under special conditions [9-12].

1. Einstein [2] has put condition on state of the body: Let there be a **stationary body** in the system (*x, y, z*), and let its energy--referred to the system (*x, y, z*) be E_0. Let the energy of the body relative to the system (ξ, η, ζ) moving as above with the velocity *v*, be H_0. The body also remains stationary in system (x, y, z) after emission of energy.

Einstein also assumed that the body also remains stationary before and after emission of light energy, which is super special condition.

But practically this condition (Light emitting body is stationary) is not obeyed in other many or numerous cases.

(i) The nuclear fission is caused by the thermal neutrons which have velocity 2,185m/s. The uranium atom also moves as it is split up in barium and krypton, and emit energy.

(ii) When a gamma ray photon of energy at least 1.02MeV, moves near the field of nucleus it is split up in electron and positron pair [35]. The gamma ray photon is in motion and so is the state of electron and positron pair.

(iii) Similarly the particle and antiparticle moves towards each other for annihilation. The particle and antiparticle collide then annihilation takes place. In nuclear fusion the atoms are set in motion. The fission is only caused by thermal neutrons (0.025eV or having velocity 2,185m/s). Thus there are characteristic or inherent conditions on the process in inter-conversion of mass and energy. These phenomena were not discovered in Einstein's time.

(iv) When a paper burns then it is also sets in motion and energy in various forms is emitted.

(v) When deuterium and titanium fuse, but only after these are set in motion under conditions of high temperature. In nuclear fusion of deuterium –tritium the energy of emitted neutrons is 14.1MeV (moving at 52,000km/s) their mass must increase about 15.36%. It may increase the mass considerably. The velocity of the reactants is not necessarily uniform and gradually they overcome the force of electrostatic repulsion. Chemical reactions were discovered in Einstein's time. Einstein never discussed this phenomenon in his works.

Thus derivation under the condition that body remains stationary in the emission process, is not conceptually useful or applicable in other case. The body remains stationary after emission of light energy, is only a theoretical perception.

Other conditions on Einstein's derivation.

Einstein's Sep. 1905 derivation [2] of $\Delta L = \Delta mc^2$ is true under *super special conditions or handpicked conditions* only. It is justified below. In the derivation of $\Delta L = \Delta mc^2$ there are FOUR variables e.g.

(a) Number of waves emitted,

(b) *l* magnitude of light energy,

(c) Angle ϕ at which light energy is emitted and

(d) Uniform velocity (relative velocity), v. The fast neutrons are slowed down and called thermal neutron thus their velocities are not necessarily uniform as can be variable while they cause fission of other nuclei.

Nature of v

According to Einstein: v is the relative velocity between system (x, y, z) and system (ξ, η, ζ). If system (x,y,z) is at rest and system (ξ, η, ζ) moves with constant velocity v, then v is relative velocity. If the system (x,y,z) and system (ξ, η, ζ) both move with same velocity then relative velocity v is zero. Further Einstein strictly took the value of velocity as uniform. The law of inter-conversion of mass and energy holds good if

(i) Velocity v is in classical region

(ii) Velocity v is in relativistic region

(iii) Velocity v is zero i.e. if both systems move with same velocity or system (ξ, η, ζ) is at rest.

(iv) Velocity v is variable or uniform

These variables have numerous values. The law of inter conversion of mass and energy holds good under all conditions, but Einstein has considered just one i.e. velocity is precisely uniform in classical region. It does not hold good under relativistic conditions. Such significant derivation must be independent of velocity.

2.1 Genuine cases neglected in Einstein's derivation

Einstein has taken super *special or handpicked* values of parameters. Thus for complete analysis the derivation can be repeated with all possible values of parameters. In all cases the law of conservation of momentum is obeyed (which is discussed in next sub-section).

(i) The body can emit large number of light waves but Einstein has taken only **TWO light waves emitted by luminous body.**

Why one or n light energy waves are neglected?

(ii) The energy of two emitted light waves may have different magnitudes but **Einstein has taken two light waves of EQUAL magnitudes (0.5L each).**

Why other magnitudes (0.500001L and 0.499999L) are neglected by Einstein?

(iii) Body may emit large number of light waves of different magnitudes of energy making different angles **(other than 0º and 180º as assumed by Einstein).**

Why other angles (such as 0º and 180.001º**,** 0.9999 º and 180º etc.) are neglected by Einstein? Thus body needs to be specially fabricated; other forms of energy such as invisible energy are not taken in account. Further body should emit light energy only, not other forms of energy.

(iv) Einstein has taken velocity in classical region (v<<c and applied binomial theorem at the end) and has not at all used velocity in relativistic region. If velocity is regarded as in relativistic region (v is comparable with c), then equation for relativistic variation of mass with velocity i.e.

$$M_{rel} = \frac{M_{rest}}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (15)$$

is taken in account. It must be noted that before Einstein's work this equation was given by Lorentz [4-5] and firstly confirmed by Kaufman [6] and afterwards more convincingly by Bucherer [7]. Einstein on June 19, 1948 wrote a letter to Lincoln Barnett [8] and advocated abandoning relativistic mass and suggested that it is better to use the expression for the momentum and energy of a body in motion, instead of relativistic mass.

It is strange suggestion as Einstein has used relativistic mass in his work including in the expression of relativistic kinetic energy [1] from which rest mass energy is derived [13-14]. So Einstein's equation of inter-conversion of mass to energy highly depends upon velocity theoretically whereas practically the mass energy inter-conversion phenomena are applicable in all cases.

(v) Einstein has considered body emits light energy, but simultaneously body may also emit heat energy which is not taken in account in Einstein's derivation. A burning body emits heat, sound, light energies and energy in form of invisible radiations simultaneously, along

with invisible radiations. For proper description of heat energy-mass inter-conversion we need equation equivalent to eq.(1). Similar is the case of other energies. In nuclear explosion energies exist in many forms e.g. light energy, sound energy, heat energy , energy in form of invisible of various radiations. Einstein has only considered Light Energy and other energies are neglected in derivation of $L = \Delta mc^2$.

Further Einstein has considered that body emits light energy in visible region. But energy can also be emitted in the invisible region and Einstein did not mention at all about heat and sound energies (emitted along with light energy). Thus energies other than light energy are also emitted but neglected by Einstein in the derivation. So energies are not taken in account completely. Thus Einstein's perception may be ideally regarded as thought experiment. In realistic scientific approach all the factors have to be taken in account to draw conclusions. The various values of parameters neglected and taken in account in derivation are shown in Table III.

Table III The values of various parameters considered by Einstein and neglected by Einstein in the derivation of Light Energy Mass equation $L = \Delta mc^2$.

Sr No	Parameters	Einstein considered	Einstein neglected (No reason was given by Einstein why parameters are neglected).
1	No. of light waves	Two Light Waves	One, three, four or n waves
2	Energy of light wave	Equal 0.5L and 0.5L each	Energies of the order of 0.500001L and 0.499999L are also possible. There are numerous such possibilities, which need to be probed. Bodies can emit more than two waves. The invisible waves of energy are not taken in account.
3	Angle	0° and 180°	The angles can be 0° and 180.001° or 0.9999° and 180° are also possible. There are numerous such possibilities which need to be probed.
4	Velocity	Classical region	The velocity can be in relativistic region. The velocity v can also be zero i.e. v = 0 v~c mass increases
5	Velocity	Uniform In classical region	The law of inter-conversion of mass to energy also holds good, when velocity is variable.

Deductions: Einstein has taken only super-special values of parameters, and neglected many realistic values.

4.0 $\Delta L \; \alpha \; \Delta mc^2$ or $\Delta L = A \; \Delta mc^2$ is equally feasible

Let in case the luminous body emits two light waves of energy 0.499999L and 0.500001L in system (x,y,z) emitted in opposite directions[10]. If waves are emitted even then body remains at rest, as velocity in this case is imperceptible i.e. 5.3×10^{-32}m/s, it can be justified on the basis of law of conservation of momentum [10,16]. Then amount of light energies measured in both systems are related as (equivalent to case of Einstein)

$E_0 = E_1 + L$ (2)

$H_o = H_1 + 0.499999 \; \beta L[(1 - v/c \cos 0^o)] + 0.500001 \; \beta \; L \; [1 - v/c \cos 180^o)]$ (38)

$(H_o - E_o) = (H_1 - E_1) + \beta \; L + 0.000002 \; \beta \; L \; v/c - L$ (39)

$(H_o - E_o) - (H_1 - E_1) = \beta \; L + 0.000002 \; \beta \; L \; v/c - L$ (40)

$= L[1 + 0.000002 \; v/c + v^2/2c^2 - 1]$ (41)

$K_b - K_a = L [1 + 0.000002 \; v/c + v^2/2c^2 - 1]$ (42)

$M_b v^2/2 - M_a v^2 /2 = L [0.000002 \; v/c + v^2/2c^2]$ (43)

$M_b - M_a = L [0.000004/cv + 1/c^2]$ (44)

$\Delta mc^2 = L[0.000004c/v + 1]$ (45)

$L = \Delta mc^2 / [0.000004c/v + 1]$ (46)

$L = \Delta mc^2 / [0.000004c/v + 1] = \Delta mc^2 / [1200+1] = \Delta mc^2 / [1201]$

$L \; \alpha \; \Delta mc^2$ or $L = A \; \Delta mc^2$ (47)

So Einstein's derivation does not give fixed value of energy corresponding to mass annihilated. Thus if Einstein's derivation is critically analysed then general result is

$L \; \alpha \; \Delta mc^2$ or $L = A \; \Delta mc^2$ (48)

where A is coefficient of proportionality.

Decrease in mass is more than L/c^2

$M_b - M_a = L [0.000004/cv + 1/c^2]$ (49)

$M_a = M_b - 0.000004L/cv - L/c^2$ (50)

Thus Einstein's derivation gives self contradictory results.

Whether the effects of recoil velocity are incorporated or not, the result remains the same as velocity of recoil is 5×10^{-33} m/s. There are numerous cases when result is $L \alpha \; \Delta mc^2$ or $L = A \; \Delta mc^2$.

4.1 Light energy has different inherent characteristics than other energies. In the derivation Einstein used eq.(1) for relativistic variation of light energy, which was speculated in the previous papers [9-12]. But this equation is only meant for light energy not at all for other energies; hence any deduction from it must be applicable for light energy only.

The equation

$$\lambda^* = \lambda \frac{\left[1 - \frac{v}{c} \cos \phi\right]}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (1)$$

is not meant for

(i) sound energy,

In Doppler effect change in frequency of sound is estimated, not variation in mass. Thus eq.(1) is not used. Likewise eq.(1) is not associated with any other energy. The speed of sound is 332m/s.

(ii) heat energy

There is no equation like eq.(1) which relates variation of heat energy. The similar is the case of other types of energies.

(iii) chemical energy

(iv) nuclear energy (v) magnetic energy (vi) electrical energy

(vii) energy emitted in form of invisible radiations

(viii) attractive binding energy of nucleus

(xi) energy emitted in cosmological and astrophysical phenomena

(x) energy emitted in volcanic reactions

(xi) energies co-existing in various forms etc. etc.

Then why results based upon eq.(1) are applied to above energies (i –xi).

The reason is that all energies have different type of nature, and the energies are not confirmed to obey the same equation.

Einstein initially derived 'light energy'–mass inter-conversion equation $L = \Delta mc^2$, then speculated 'every energy' –mass inter conversion equation $E = \Delta mc^2$ from $L = \Delta mc^2$. As eq. (1) is only meant for light energy, not for other energies. Hence speculative transition to $E = \Delta mc^2$ from $L = \Delta mc^2$ is absolutely without any mathematical basis.

4.2 If the measuring system is at rest (v=0) and body emits two light waves as in Einstein's derivation then derivation is not applicable. v can also zero if system (x,y,z) and system (ξ, η, ζ) move with same velocity.

However in this case experimentally when light energy is emitted mass decreases. It is serious limitation of Einstein's derivation.

When the measuring system (ξ, η, ζ) is at rest v = 0 then

$$\lambda^* = \lambda \qquad (51)$$

$$H_o = H_1 + L/2 + L/2 \qquad (52)$$

$$E_o = E_1 + L \qquad (2)$$

$$(H_o - E_o) - (H_1 - E_1) = 0 \qquad (53)$$

As body is at rest and measuring system **(ξ, η, ζ)** is also at rest, then $(H_o - E_o)$ or $(H_1 - E_1)$ cannot be interpreted as kinetic energy. Hence further derivation is not applicable. This is mathematically critical analysis, have no implications on experimentally established status of $\Delta L = \Delta mc^2$ ($E = \Delta mc^2$).

4.3 Superluminal speed of neutrinos: In addition experiments are being conducted to measure the velocity (may exceeds more than c) of neutrinos in MIMOS+ experiments at Fermi Laboratory. The experiment will aim to measure the time a neutrino needs to travel from Fermilab to the Soudan mine in Minnesota with a precision of about 1nanosecond [15] . So it far more improved of experiments conducted 2007 known as MINOS. If speed of neutrinos is found even equal to c , this discussion will be further validated.

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