

New proof of general relativity through the correct physical interpretation of the Mössbauer rotor experiment

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Abstract

In this Essay, we give a correct interpretation of a historical experiment by Kündig on the transverse Doppler shift in a rotating system (Mössbauer rotor experiment). This experiment has been recently first reanalyzed, and then replied by an experimental research group. The results of reanalyzing the experiment have shown that a correct re-processing of Kündig's experimental data gives an interesting deviation of a relative redshift between emission and absorption resonant lines from the standard prediction based on the relativistic dilatation of time. Subsequent new experimental results by the reply of Kündig experiment have shown a deviation from the standard prediction even higher. By using the Equivalence Principle (EP), which states the equivalence between the gravitational "force" and the *pseudo-force* experienced by an observer in a non-inertial frame of reference (included a rotating frame of reference), here the theoretical framework of the Mössbauer rotor experiment is reanalyzed directly in the rotating frame of reference through a general relativistic treatment. It will be shown that previous analyses missed an important effect of clock synchronization. By adding this new effect, the correct general relativistic prevision is in perfect agreement with the new experimental results. Such an effect of clock synchronization has been missed in various papers in the literature, with some subsequent claim of invalidity of the relativity theory and/or some attempts to explain the experimental results through "exotic" effects. The general relativistic interpretation in this Essay shows, instead, that the new experimental results of the Mössbauer rotor experiment are a new, strong and independent, proof of general relativity.

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To the memory of Enrico Lista.

We give a correct interpretation of a historical experiment by Kündig on the transverse Doppler shift in a rotating system, measured with the Mössbauer effect (Mössbauer rotor experiment) [3]. The Mössbauer effect (discovered by R. Mössbauer in 1958 [14]) consists in resonant and recoil-free emission and absorption of gamma rays, without loss of energy, by atomic nuclei bound in a solid. It resulted and currently results very important for basic research in physics and chemistry. In this Essay, we will focus on the so called Mössbauer rotor experiment. In this particular experiment, the Mössbauer effect works through an absorber orbited around a source of resonant radiation (or vice versa). The aim is to verify the relativistic time dilation for a moving resonant absorber (the source), inducing a relative energy shift between emission and absorption lines.

In a couple of recent papers [1, 2], the authors first reanalyzed in [1] the data of a known experiment of Kündig on the transverse Doppler shift in a rotating system, measured with the Mössbauer effect [3], and second, they carried out their own experiment on the time dilation effect in a rotating system [2]. In [1], it has been found that the original experiment by Kündig [3] contained errors in the data processing. A puzzling fact was that, after correction of the errors of Kündig, the experimental data gave the value [1]

$$\frac{\nabla E}{E} \simeq -k \frac{v^2}{c^2}, \quad (1)$$

where $k = 0.596 \pm 0.006$, instead of the standard relativistic prediction $k = 0.5$ due to time dilatation. The authors of [1] stressed that the deviation of the coefficient k in equation (1) from 0.5 exceeds by almost 20 times the measuring error and that the revealed deviation cannot be attributed to the influence of rotor vibrations and other disturbing factors. All these potential disturbing factors have been indeed excluded by a perfect methodological trick applied by Kündig [3], that is a first-order Doppler modulation of the energy of γ -quanta on a rotor at each fixed rotation frequency. In that way, Kündig's experiment can be considered as the most precise among other experiments of the same kind [4–8], where the experimenters measured only the count rate of detected γ -quanta as a function of rotation frequency. The authors of [1] have also shown that the experiment [8], which contains much more data than the ones in [4–7], also confirms the supposition $k > 0.5$. Motivated by their results in [1], the authors carried out their own experiment [2]. They decided to repeat neither the scheme of the Kündig experiment [3], nor the schemes of other known experiments on the subject previously mentioned above [4–8]. In that way, they got independent information on the value of k in equation (1). In particular, they refrained from the first-order Doppler modulation of the energy of γ -quanta, in order to exclude the uncertainties in the realization of this method [2]. They followed the standard scheme [4–8], where the count rate of detected γ -quanta N as a function of the rotation frequency ν is measured. On the other hand, differently from the experiments [4–8], they evaluated the influence of chaotic

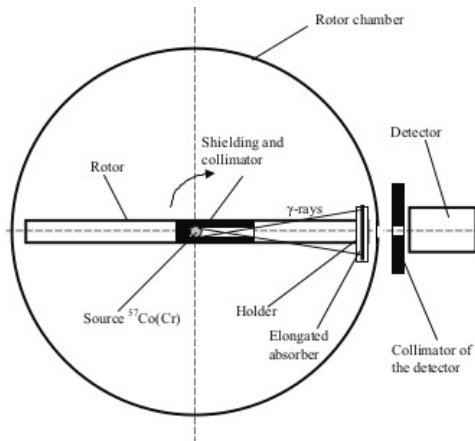


Figure 1: Scheme of the new Mössbauer rotor experiment, adapted from ref. [2]

vibrations on the measured value of k [2]. Their developed method involved a joint processing of the data collected for two selected resonant absorbers with the specified difference of resonant line positions in the Mössbauer spectra [2]. The result obtained in [2] is $k = 0.68 \pm 0.03$, confirming that the coefficient k in Eq. (1) substantially exceeds 0.5. The scheme of the new Mössbauer rotor experiment is in Figure 1, while technical details on it can be found in [2].

In this Essay, the EP, which states the equivalence between the gravitational "force" and the *pseudo-force* experienced by an observer in a non-inertial frame of reference (included a rotating frame of reference), will be used to reanalyze the theoretical framework of Mössbauer rotor experiments directly in the rotating frame of reference, by using a full general relativistic treatment [16]. The results will show that previous analyses missed an important effect of clock synchronization and that the correct general relativistic prevision gives $k \simeq \frac{2}{3}$ [16], in perfect agreement with the new experimental results of [2]. In that way, the general relativistic interpretation of this Essay shows that the new experimental results of the Mössbauer rotor experiment are a new, strong and independent, proof of general relativity. We also stress that various papers in the literature (included ref. [4] published in Phys. Rev. Lett.) missed the effect of clock synchronization [1–8], [11–13] with some subsequent claim of invalidity of relativity theory and/or some attempts to explain the experimental results through "exotic" effects [1, 2, 11, 12, 13].

Following [9, 16], one considers a transformation from an inertial frame, in which the space-time is Minkowskian, to a rotating frame of reference. Using cylindrical coordinates, the line element in the starting inertial frame is [9, 16]

$$ds^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 - dz^2. \quad (2)$$

The transformation to a frame of reference $\{t', r', \phi', z'\}$ rotating at the uniform angular rate ω with respect to the starting inertial frame is given by [9, 16]

$$t = t' \quad r = r' \quad \phi = \phi' + \omega t' \quad z = z' . \quad (3)$$

Thus, Eq. (2) becomes the following well-known line element (Langevin metric) in the rotating frame [9, 16]

$$ds^2 = \left(1 - \frac{r'^2 \omega^2}{c^2}\right) c^2 dt'^2 - 2\omega r'^2 d\phi' dt' - dr'^2 - r'^2 d\phi'^2 - dz'^2. \quad (4)$$

The transformation (3) is both simple to grasp and highly illustrative of the general covariance of general relativity as it shows that one can work first in a "simpler" frame and then transforming to a more "complex" one [16]. As one considers light propagating in the radial direction ($d\phi' = dz' = 0$), the line element (4) reduces to [16]

$$ds^2 = \left(1 - \frac{r'^2 \omega^2}{c^2}\right) c^2 dt'^2 - dr'^2. \quad (5)$$

The EP permits to interpret the line element (5) in terms of a curved spacetime in presence of a static gravitational field [10, 15, 16]. In that way, one obtains a purely general relativistic interpretation of the pseudo-force experienced by an observer in a rotating, non-inertial frame of reference [16]. Setting the origin of the rotating frame in the source of the emitting radiation, one gets a first contribution, which arises from the "gravitational redshift", that can be directly computed using Eq. (25.26) in [10], which, in the twentieth printing 1997 of [10], is written as

$$z \equiv \frac{\Delta\lambda}{\lambda} = \frac{\lambda_{received} - \lambda_{emitted}}{\lambda_{emitted}} = |g_{00}(r'_1)|^{-\frac{1}{2}} - 1 \quad (6)$$

and represents the redshift of a photon emitted by an atom at rest in a gravitational field and received by an observer at rest at infinity. Here, a slightly different equation with respect to Eq. (25.26) in [10] will be used, because here one considers a gravitational field which increases with increasing radial coordinate r' , while Eq. (25.26) in [10] concerns a gravitational field which decreases with increasing radial coordinate [16]. Also, the zero potential is set in $r' = 0$ instead of at infinity, and one uses the proper time τ instead of the wavelength λ [16]. Thus, by using Eq. (5), one gets [16]

$$\begin{aligned} z_1 &\equiv \frac{\nabla\tau_{10} - \nabla\tau_{11}}{\tau} = 1 - |g_{00}(r'_1)|^{-\frac{1}{2}} = 1 - \frac{1}{\sqrt{1 - \frac{(r'_1)^2 \omega^2}{c^2}}} \\ &= 1 - \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \simeq -\frac{1}{2} \frac{v^2}{c^2}, \end{aligned} \quad (7)$$

where $\nabla\tau_{10}$ is the delay of the emitted radiation, $\nabla\tau_{11}$ is the delay of the received radiation, $r'_1 \simeq c\tau$ is the radial distance between the source and the detector and $v = r'_1 \omega$ is the tangential velocity of the detector [16]. Hence, one finds a first contribution, say $k_1 = \frac{1}{2}$, to k [16]. We stress again that the power of the EP

enabled us to use a pure general relativistic treatment in the above discussion [16].

Now, one notices that the variations of proper time $\nabla\tau_{10}$ and $\nabla\tau_{11}$ have been calculated in the origin of the rotating frame which is located in the source of the radiation [16]. But the detector is moving with respect to the origin in the rotating frame [16]. Thus, the clock in the detector must be synchronized with the clock in the origin, and this gives a second, additional, contribution to the redshift [16], which was missed in previous analyses [1–8], [11–13]. To compute this second contribution, one uses Eq. (10) of [9], which represents the proper time increment $d\tau$ on the moving clock having radial coordinate r' for values $v \ll c$

$$d\tau = dt' \left(1 - \frac{r'^2 \omega^2}{c^2} \right). \quad (8)$$

Inserting the condition of null geodesics $ds = 0$ in Eq. (5), one gets [16]

$$cdt' = \frac{dr'}{\sqrt{1 - \frac{r'^2 \omega^2}{c^2}}}, \quad (9)$$

where the positive sign in the square root has been taken, because the radiation is propagating in the positive r direction [16]. Combining eqs. (8) and (9), one obtains [16]

$$cd\tau = \sqrt{1 - \frac{r'^2 \omega^2}{c^2}} dr'. \quad (10)$$

Eq. (10) is well approximated by [16]

$$cd\tau \simeq \left(1 - \frac{1}{2} \frac{r'^2 \omega^2}{c^2} + \dots \right) dr', \quad (11)$$

which permits to find the second contribution of order $\frac{v^2}{c^2}$ to the variation of proper time as [16]

$$c\nabla\tau_2 = \int_0^{r'_1} \left(1 - \frac{1}{2} \frac{(r'_1)^2 \omega^2}{c^2} \right) dr' - r'_1 = -\frac{1}{6} \frac{(r'_1)^3 \omega^2}{c^2} = -\frac{1}{6} r'_1 \frac{v^2}{c^2}. \quad (12)$$

Thus, as $r'_1 \simeq c\tau$ is the radial distance between the source and the detector, one gets the second contribution of order $\frac{v^2}{c^2}$ to the redshift as [16]

$$z_2 \equiv \frac{\nabla\tau_2}{\tau} = -k_2 \frac{v^2}{c^2} = -\frac{1}{6} \frac{v^2}{c^2}. \quad (13)$$

Then, one obtains $k_2 = \frac{1}{6}$ and, using eqs. (7) and (13), the total redshift is [16]

$$\begin{aligned} z &\equiv z_1 + z_2 = \frac{\nabla\tau_{10} - \nabla\tau_{11} + \nabla\tau_2}{\tau} = -(k_1 + k_2) \frac{v^2}{c^2} \\ &= -\left(\frac{1}{2} + \frac{1}{6}\right) \frac{v^2}{c^2} = -k \frac{v^2}{c^2} = -\frac{2}{3} \frac{v^2}{c^2} = 0.6 \frac{v^2}{c^2}, \end{aligned} \quad (14)$$

which is completely consistent with the result $k = 0.68 \pm 0.03$ in [2].

We stress that the additional factor $-\frac{1}{6}$ in Eq. (13) comes from clock synchronization [16]. In other words, its theoretical absence in the works [1–8], [11–13] reflected the incorrect comparison of clock rates between a clock at the origin and one at the detector [16]. This generated wrong claims of invalidity of relativity theory and/or some attempts to explain the experimental results through “exotic” effects [1, 2, 11, 12, 13] which, instead, must be rejected. Notice that, even in discussing the effect of clock synchronization, a pure general relativistic treatment has been performed.

The appropriate reference [9] has been evoked for a discussion of the Langevin metric. This is dedicated to the use of general relativity in Global Positioning Systems (GPS), which leads to the following interesting realization [16]: the correction of $-\frac{1}{6}$ in Eq. (13) is analogous to the correction that one must consider in GPS when accounting for the difference between the time measured in a frame co-rotating with the Earth geoid and the time measured in a non-rotating (locally inertial) Earth centered frame (and also the difference between the proper time of an observer at the surface of the Earth and at infinity). Indeed, if one simply considers the gravitational redshift due to the Earth’s gravitational field, but neglects the effect of the Earth’s rotation, GPS would not work [16]! The key point is that the proper time elapsing on the orbiting GPS clocks cannot be simply used to transfer time from one transmission event to another because path-dependent effects must be taken into due account, exactly like in the above discussion of clock synchronization [16]. In other words, the obtained correction $-\frac{1}{6}$ in Eq. (13) is not an obscure mathematical or physical detail, but a fundamental ingredient that must be taken into due account [16]. Further details on the analogy between the results of this Essay and the use of general relativity in GPS have been highlight in [16].

Conclusion remarks

In this Essay, the power of the EP, which states the equivalence between the gravitational "force" and the *pseudo-force* experienced by an observer in a non-inertial frame of reference (included a rotating frame of reference), has been used to reanalyze, from a pure general relativistic point of view, the theoretical framework of the new Mössbauer rotor experiment in [2], directly in the rotating frame of reference. The results have shown that previous analyses missed an important effect of clock synchronization and that the correct general relativistic prevision gives $k \simeq \frac{2}{3}$, in perfect agreement with the new experimental results in [2]. Thus, in this Essay it has been shown that the general relativistic interpretation of the new experimental results of the Mössbauer rotor experiment is a new, strong and independent, proof of Einstein general relativity. The importance of the results in this Essay is stressed by the issue that various papers in the literature (included ref. [4] published in Phys. Rev. Lett.) missed the effect of clock synchronization [1–8], [11–13], with some subsequent claim of invalidity of relativity theory and/or some attempts to explain the experimental results through “exotic” effects [1, 2, 11, 12, 13], which, instead, must be rejected.

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References

- [1] A. L. Kholmetskii, T. Yarman and O. V. Missevitch, *Phys. Scr.* **77**, 035302 (2008).
- [2] A. L. Kholmetskii, T. Yarman, O.V. Missevitch and B. I. Rogozev, *Phys. Scr.* **79**, 065007 (2009).
- [3] W. Kündig, *Phys. Rev.* **129**, 2371 (1963).
- [4] H. J. Hay et al, *Phys. Rev. Lett.* **4**, 165 (1960).
- [5] H. J. Hay, in *Proc. 2nd Conf. Mössbauer Effect*, ed A Schoen and D M T Compton (New York: Wiley) p 225 (1962).
- [6] T. E. Granshaw and H. J. Hay, in *Proc. Int. School of Physics, ‘Enrico Fermi’* (New York: Academic) p 220 (1963).
- [7] D. C. Champeney and P. B. Moon, *Proc. Phys. Soc.* **77**, 350 (1961).
- [8] D. C. Champeney, G. R. Isaak and A. M. Khan, *Proc. Phys. Soc.* **85**, 583 (1965).
- [9] N. Ashby, *Liv. Rev. Rel.* **6**, 1 (2003).
- [10] C. W. Misner , K. S. Thorne, J. A. Wheeler, “*Gravitation*”, Feeman and Company (1973).
- [11] A. Kholmetskii, T. Yarman, O. Missevitch, *Eur. Phys. Jour. Plus*, 128, 42 (2013).
- [12] V. O. de Haan, *Jour. Comp. Meth. Scien. Eng.*, 13, 51 (2013).
- [13] A. L. Kholmetskii, T. Yarman, M. Arik and O.V. Missevitch, *AIP Conf. Proc.* 1648, 510011 (2015).
- [14] R. L. Mössbauer, *Zeitschrift für Physik A* (in German) 151, 124 (1958).
- [15] L. Landau and E. Lifshits, *Classical Theory of Fields* (3rd ed.), London: Pergamon (1971).
- [16] C. Corda, *Ann. Phys.* 355, 360 (2015).