LAGRANGE: how to measure the angular momentum of the galactic dark halo

Author: Angelo Tartaglia, DISAT, Politecnico di Torino and INdAM

Short summary

It is well known that general relativity predicts the presence of a gravito-magnetic component of the gravitational interaction induced by a rotating mass. Such contribution has been verified for the angular momentum of the earth. Here we present a proposal to measure the gravito-magnetism of the sun and possibly of the dark galactic halo currently assumed to incorporate our galaxy (as well as the others). The experimental approach is based on the fully relativistic version of the Sagnac effect. When an electromagnetic signal is led to travel along a spacely closed path and the material device with which the beam interacts rotates with respect to the “fixed stars”, the time of flight for a complete turn depends on the direction of rotation. The same happens when the loop is immersed in a gravito-magnetic field, even if it is not rotating: this is the typical Lense-Thirring drag. In this case the asymmetry in the time of flight is proportional to the projection of the angular momentum of the main body onto an axis perpendicular to the plane of the loop. This behavior is the basis of measurements using ringlasers. Going to space and having the sun and its angular momentum as a target, the implementation of a “Sagnac” approach is still possible but the size of the closed circuit must be at the scale of the orbit of the earth. Furthermore the geometry of the loop needs be reasonably stable. The solution we propose is to exploit the Lagrange points of the sun-earth system. The configuration of the Lagrange points is indeed stable (if not strictly rigid) and moves around together with the earth. Considering as an example the triangle L_2–L_4–L_5 (according to the standard enumeration of the points) the expected time of flight asymmetry due to the angular momentum of the sun is approximately 4×10^{-13} s. Other combinations of the *L*-points are of course possible, but the order of magnitude of the effect is more or less the same. In this case the technique cannot be the same as for ringlasers, rather a local measurement of the proper time interval between the arrivals of “right” and “left” signals is required. The same approach lends also an interesting opportunity to verify the presence or absence of an effect originating from the dark halo of our galaxy. In fact if the halo is there and it interacts gravitationally with the visible matter of the Milky Way, it has also to rotate with the same peripheral speed as the stars. If, then, the halo is much more massive than the baryonic matter its angular momentum too has to be large and, if general relativity is true, it must produce a gravito-magnetic field. To work out numbers requires the knowledge or reasonable guesses about the mass distribution of the dark matter, but the very nature of that matter is unimportant and the scale of the halo and its total mass tell us that the effect could be non-negligible. An advantage of the Sagnac-like approach, worth mentioning, is that it neutralizes the dominant effect of the gravito-electric component of the field, since the latter has no chiral symmetry.

Use of the sun-earth Lagrangian points L_1, L_2, L_4, L_5, together with the earth, for:
a) establishing the base for a relativistic positioning and navigation system within the solar system;
b) general relativity (GR) tests and deviations from GR, especially related to the gravito-magnetic field of the sun (solar Lense-Thirring effect);
c) accurate measurements of deflection of light by the sun;
d) establishment of constraints on extended or alternative (to GR) theories of gravity, better than the existing ones.
The advantages of a reference frame based on the Lagrangian points is that it is “rigidly” rotating together with the earth while all points are in free fall (at the Newtonian approximation level).

Placing pulsed emitters of electromagnetic signals (artificial pulsars) in the L points would allow the relativistic selfpositioning and selfguidance of spacecrafts within the solar system, by means of time measurements performed on board (arrival times sequences of the pulses at the user's position). The fact that the four beacons (plus earth) are at rest with respect to each other and the mutual distances are in the order of 10^{11} m leads to a very simple algorithm for working out the position of the receiver. A very high geometric accuracy of the positioning depending, in principle, only on the accuracy of the onboard clock, would be attained.

On the scientific side, the size of the Lagrangian loops allows an accurate measurement of the solar Lense-Thirring effect by means of time of flight measurements. Electromagnetic pulses sent around along a complete Lagrangian contour (circulating from an L point to the next until coming back to the start) would take a few thousands seconds time of flight (tof ~2000 s). The left/right tof difference induced by the angular momentum of the sun, considering the size of the loop, is of the order of 10^{-13} s. A precision for the measurement of the tof in the order of 10^{-15} s would provide a 1% detection of the solar Lense-Thirring. The stability required to the measuring clock, within the precision 10^{-15} s (or better), would be over a duration longer than the round trip tof, i.e. over a half hour or so. The measurement can be repeated many times (~1000 times/year) providing a statistical basis to tackle problems related with the movement of the emitters around the L points. The full mission may be deployed gradually and include already flying missions, especially GAIA, located in L_2; the latter is endowed with communication channels that can be exploited for transmitting signals in a closed loop including the spacecraft.

The inclusion of GAIA and its data in the Lagrange network would allow to exploit GAIA's time component of the local-line-of-sight, as defined in the GR measurement theory and modelled in the observables of the mission.

Altogether, the total number of receiving/emitting stations at completion of the project would be five (including the earth), which means that intermediate stages may be realized, composed of one new station in the sky plus GAIA and the earth, two stations in the sky plus GAIA and the earth… In the final configuration redundancy is available, using all closed loops among the five stations.

The deployment of the system (possibly with a collaboration among space agencies, accepting to accomplish separate missions) would lend the opportunity for a number of additional experiments performed at little additional cost.

Optical monitoring of the apparent position of the spacecraft (SC) in L_4 and L_5 with respect to the background stars (and viceversa) and using L_2 as a calibration position (reference triangle L_2-L_4-L_5), provides an opportunity for highly accurate measurements of the deflection of light in the field of the sun. For this use the SC in the three cited Lagrangian points would be equipped with both radio and light transponder devices in order to provide optical beacons for visual localization from telescopes on earth. The combination of a deflection ~7 mas (milliarcsecond) and an accuracy in the positioning ~0.1 m would give a precision in the evaluation of the deflection ~10^{-5}, one order of magnitude better than what achievable by direct angle measurements only, even at the µas level. The frequency dependence of the deflection would also be explored.

Accurately tracking the orbit of spacecraft around L_1 and L_2 can be used to test the Strong Equivalence Principle (SEP), posing constraints on the η violation parameter.

Tracking the orbit of the SCs around the Lagrangian points would also allow for measurement of the gravitational potential around saddle points. It would then be possible to evidence non linear effects predicted by MOND and connected with the dark energy, if they exist. It would as well as be possible to address the issue of the presence or absence of Yukawa like terms in the gravitational interaction, assessing upper limits for the size of the corresponding Yukawa parameter.

References

A. Tartaglia, E. C. Lorenzini, D. Lucchesi, G. Pucacco, M. L. Ruggiero, P. Valko, *Gen Relativ Gravit* (2018) 50:9