Caracteristics of Black Hole in Loop Quantum Gravity

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We will present recent results on black holes in effective loop quantum gravity. Quantum gravity effects might allow the transition of a black hole into a white hole, when the Planck density is reached. In this talk, I will briefly review previous studies and focus on the random nature of the bouncing lifetime which has not yet been taken into account. I will show that, when we consider a stochastic lifetime, the signal emitted by bouncing black holes might explain the fast radio bursts. Then, I will present recent results on the emission cross section calculated for a quasi Schwarzschild black hole including loop quantum gravity corrections. Indeed, the black hole geometry deformation by quantum effect has consequences for cross section and Hawking spectrum.

Keywords: Black Hole, Quantum Gravity

1. Bouncing Black Hole[1]

A bouncing black hole is described by a classical collapsing solution which is linked to the classical exploding one by a quantum tunneling. It's argue[2 3] that a black hole of masse M would have a lifetime of the order of M^2. On account of the tunneling process, the lifetime of a black hole should be considered as a random variable. The main lifetime of a black hole with a mass M is $\tau = kM^2$, with k chosen to be of the order of 0.05[2]. The probability that a black hole has not yet bounced after a time t is given by $P(t) = \frac{1}{\tau}e^{-\frac{t}{\tau}}$. This is like the usual nuclear decay behavior. We focus on local effects and we considered primordial black holes (PBHs) because we are interesting in black holes which bounce in the contemporary universe. The number of black holes bouncing after the Hubble time t_H in a time interval dt is:

$$dN = \frac{N_0}{kM^2}e^{-\frac{t_H}{kM^2}}dt, \tag{1}$$

where N_0 is the initial abundance. The initial differential mass spectrum of the considered PBHs is given by dN/dM. Photons emitted have a characteristic wavelength of the order of the size of the black hole (the unique lenght scale of the problem). We model the shape of the signal emitted by a single black hole by a Gaussian function:

$$\frac{dN_\gamma^{BH}}{dE} = Ae^{-\frac{(E-E_0)^2}{2\sigma_E^2}}, \tag{2}$$

where $E_0 = 1/(2R_S) = 1/(4M)$, R_S is the Schwarzschild radius. The width is fixed to be $\sigma_E = 0.1E_0$ but the results do not critically depend on this value. The full signal due to a local distribution of bouncing black holes is given by

$$\frac{dN_\gamma}{dE} = \int_{M_{Pl}}^{\infty} Ae^{-\frac{(E-E_0)^2}{2\sigma_E^2}} \cdot \frac{dN}{dM}(M) \cdot \frac{1}{kM^2} e^{-\frac{t_H}{kM^2}}. \tag{3}$$

We considered two types of mass spectrum for the PBHs: a peaked one (4) (from[4] for example) centered around a value M_0 and a wide one (5) (from[5]).

$$\frac{dN}{dM} \propto e^{-\frac{(M-M_0)^2}{2\sigma_M^2}}, \tag{4}$$

$$\frac{dN}{dM} \propto M^\alpha. \tag{5}$$

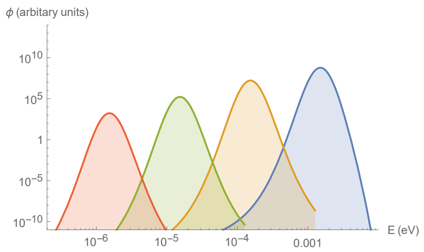

Fig. 1. Differential electromagnetic flux emitted by bouncing PBHs for a central mass M_0 equal (from right to left) to M_{t_H}, $10M_{t_H}$, $100M_{t_H}$, and $1000M_{t_H}$.

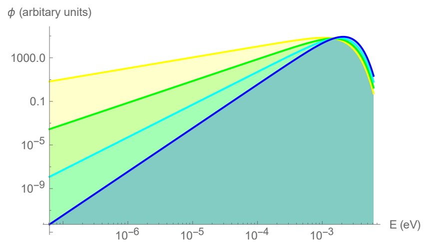

Fig. 2. Signal expected from a wide mass spectrum, with $\alpha = \{-3, -2, -1, 0\}$ from the lower curve to the upper curve at 10^{-6} eV.

In Fig 1, the expected emitted flux is shown for different values of the central mass M_0 where M_{t_H} is the mass satisfying $t_H = kM_{t_H}^2$. Because of the stochastic process, the mean energy of the emitted signal can be different from the determined one considered in previous studies. If the mass spectrum is peaked around masses higher than M_{t_H}, it is perfectly possible to precisely account for the typical wavelength of FRBs. Indeed the curve on the left in Fig 1 is peaked around 1.5 GHz. In Fig 2 we present the expected signal for a wide spectrum. The shape of the mass spectrum does influence the expected signal (not only the lifetime) as the probabilistic nature of the lifetime is now taken into account: black holes with masses smaller or larger than M_{t_H} do also contribute to the emitted radiation and changing their relative weights does change the result. This wide spectrum predict that one should expect a higher flux as the energy increases (up to the infrared band). The slope of this increase reflects that of the mass spectrum. This is qualitatively quite independent of the details of the mass spectrum.

The key point of this study was to show that the randomness of the lifetime of black holes in quantum gravity can drastically change the spectral characteristic of the expected signal and can lead to predictions.

2. Emission Cross section for Loop Black Hole

Hawking evaporation[6] results from the description of a quantum field in a classical curve space-time. A far external observer would see a particles emission with a blackbody spectrum at temperature $T_H = 1/(8\pi M)$, with M the black hole mass. One considering the gravitational and centrifugal potentials, the thermal distribution is multiplying by the emission cross section such that the emitted flux is:

$$\frac{dN}{dt} = \frac{1}{e^{\frac{\omega}{T_H}} \pm 1}\sigma(M,s,\omega)\frac{d^3k}{(2\pi)^3}, \tag{6}$$

with s the particle spin and w its energy. We calculate the cross section for a loop black hole: it is a quasi Schwarzschild black hole which take into account of quantum effects of Loop Quantum Gravity describing by the following metric[7]:

$$ds^2 = -G(r)dt^2 + \frac{dr^2}{F(r)} + H(r)d\Omega^2, \tag{7}$$

$$G(r) = \frac{(r-r_+)(r-r_-)(r+r_*)^2}{r^4+a_0^2} \tag{8}$$

$$F(r) = \frac{(r-r_+)(r-r_-)r^4}{(r+r_*)^2(r^4+a_0^2)} \tag{9}$$

$$H(r) = r^2 + \frac{a_0^2}{r^2} \tag{10}$$

with $a_0 = \frac{\sqrt{3}\gamma l_{Pl}^2}{2}$, $r_+ = 2M$, $r_- = 2MP^2$, $r_* = \sqrt{r_+r_-} = 2MP$ and $d\Omega^2 = d\theta^2 + sin^2\theta d\phi^2$. γ is the Barbero-Immirzi parameter, $P = (\sqrt{1+\epsilon^2}-1)$ is the polymeric function, $\epsilon = \gamma\delta$, with δ the polymeric parameter. When $\delta \to 0$, (7) tends to the Schwarzschild metric. According to the optical theorem[8], the latter is related to the transmission coefficient for the mode l, A_l, by:

$$\sigma(\omega) = \sum_{l=0}^{\infty}\frac{(2l+1)\pi}{\omega^2}|A_l|^2, \tag{11}$$

2.1. *Massless scalar field*

Considering symmetries, the scalar field can be written as $\Phi(r,\theta,\phi,t) = R(r)A(\theta)e^{i(wt+m\phi)}$. Its dynamics in a gravitationnal field is described by the generalized Klein-Gordon equation (12) and with the metric (7) we obtain the radial equation (13):

$$\frac{1}{\sqrt{-g}}\partial_\mu(g^{\mu\nu}\sqrt{-g}\partial_\nu\Phi) = 0, \tag{12}$$

$$\frac{\sqrt{GF}}{H}\partial_r\left(H\sqrt{GF}\partial_r R\right) + VR = 0, \tag{13}$$

with $V = \left(\omega^2 - \frac{G}{H}l(l+1)\right)$ and l the orbital quantum number. At the horizon r_+, the radial part of the wave function R^h will be a plane wave (14) with respect to the tortoise coordinate r^* defined as $dr^{*2} \equiv \frac{dr^2}{GF}$. Infinitly far, the radial wave function R^∞ is a spherically wave (15).

$$R^h(r^*) = A_{in}^h e^{-i\omega r^*} + A_{out}^h e^{i\omega r^*} \tag{14}$$

$$R^\infty(r) = \frac{A_{in}^\infty}{r}e^{-i\omega r} + \frac{A_{out}^\infty}{r}e^{i\omega r} \tag{15}$$

with A_{in}^i the probabilty amplitude for incoming mode and A_{out}^i the probabilty amplitude for outcoming mode.

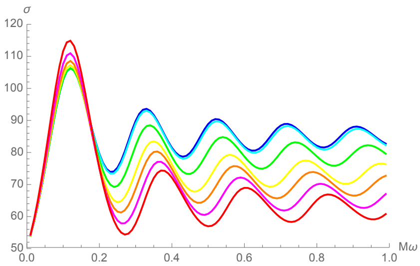

Figure 3 : Emission cross section for a scalar field with an energy ω in a loop black hole with a mass M for different value of ϵ. From bottom to top we have $\epsilon = 10^{\{0.2,0.1,0,-0.1,-0.3,-0.8,-3\}}$. The blue one is superposed with the cross section of a Schwarzschild black hole.

From the asymptotic solutions we calculate the transmission amplitude:

$$|A_l|^2 = 1 - \left|\frac{A_{out}^{\infty}}{A_{in}^{\infty}}\right|^2 , \tag{16}$$

and then we deduce the cross section. As far the metric 7 tends to the Schwarzschild solution when ϵ tends to zero, we observe the same behavior for the cross section. For $\epsilon < 10^{-0.8}$ we can't distinguished the two solutions. We conclude that taken into account of the quantum correction does not influence the cross section of a scalar field for reasonnable value of ϵ, that is to say $\epsilon << 1$.

2.2. *Spin $\frac{1}{2}$ field*

For spin $\frac{1}{2}$ field, we use the Newman-Penrose formalism[9] and pursue the Chandrasekhar procedure[10] to obtain the following radial equation:

$$\sqrt{2H}\left(\mathcal{D}^{\dagger}\left(\frac{\sqrt{2H}\mathcal{D}R(r)}{\lambda - i\mu_*\sqrt{2H}}\right) - i\mu_* R(r)\right) = \lambda R(r) \tag{17}$$

with $\mathcal{D} = \frac{\sqrt{F}}{2\sqrt{2}}\left(\frac{\partial_r H}{H} + \frac{\partial_r G}{2G}\right) + \frac{\sqrt{F}}{\sqrt{2}}\partial_r - \frac{i\omega}{\sqrt{2G}}$ and $\lambda^2 = (l+\frac{1}{2})^2$. Computation are in progress, the results would be presented in an on going redaction article.

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